House Price Extrapolation, Debt, and Monetary Policy: An Analytical Approach

– preliminary and incomplete –

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Abstract

This paper introduces a tractable heterogeneous agent New Keynesian (HANK) model with a housing sector and house price belief extrapolation. The model consists of a saver and a borrower, where the borrower is hand-to-mouth and uses housing as collateral. We identify four main channels through which house price extrapolation affects borrowers' demand: an indirect channel through aggregate demand, a direct collateral channel, a precautionary savings motive, and a fire sale motive. Turning to monetary policy we show that the central bank faces a trade-off between stabilizing borrower consumption and house prices (i.e., saver consumption). Since only one policy tool is available, both cannot be stabilized simultaneously, making monetary policy alone suboptimal tool in this context.

JEL Codes: E13, E21, E32, E70

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I. Introduction

Housing is arguably the most significant asset for households, serving not only as a durable consumption good but also as a critical form of collateral in debt markets. At the same time, house prices are subject to boom-bust cycles, driven by over-optimism and over-pessimism, which generate considerable volatility in housing markets.[1] The interaction between these two characteristics—price volatility and its impact on households' borrowing capacity—has profound macroeconomic implications. A decline in house prices constrains borrowers ability to attain credit which leads to a decline in borrower consumption. Simultaneously, falling house prices foster pessimism regarding future price developments, incentivizing households to sell, thereby exerting additional downward pressure on prices. This further tightens borrowing constraints, creating a self-reinforcing cycle of declining prices and reduced access to credit. In such periods, policy intervention, particularly monetary policy, becomes essential in counteracting this downward spiral.

This paper contributes to the literature by introducing a tractable heterogeneous agents New Keynesian (HANK) model that incorporates a housing sector and house price extrapolation. Within this framework, we first characterize the key channels through which house price extrapolation interacts with borrowers' demand for goods. Four main channels are identified: an indirect channel operating through aggregate demand, a direct channel stemming from housing as collateral, a precautionary savings motive, and a fire sale motive.

We then examine potential monetary policy options and demonstrate that the monetary authority faces a trade-off between stabilizing borrower consumption and house prices, i.e. saver consumption. Given that the policy authority has only a single tool at its disposal, it is unable to simultaneously stabilize both. As a result, monetary policy alone is not an optimal instrument for addressing these types of situations.

Our model environment is a typical tractable HANK model, as in Bilbiie (2024), in which we introduce a housing sector. Borrowers, are hand-to-mouth (HtM), and they borrower from savers against the housing units the hold. House price beliefs are formed by extrapolating observed house price growth into the future. The economy faces to

[1] See e.g.: Case et al. (2012); Armona et al. (2019); Kuchler and Zafar (2019); Ma (2020); Kaplan et al. (2020)

shocks, a shock to the borrowing constraint, and a house price belief shock.

In this framework, using certain simplifying assumptions, we demonstrate that the model features two key channels that link house prices to borrowers.[2] First, house prices *directly* impact borrowers' ability to access credit. Second, house prices influence aggregate demand and, consequently, borrowers' income—an *indirect* effect typical of many standard HANK models. In this context, house price extrapolation serves to amplify price volatility.

Monetary policy influences the economy in two distinct ways: it *contemporaneously* affects house prices, and thus the current state of the economy, while also determining the future rate at which debt will be repaid, introducing effects with a *lag*. In response to the two shocks under consideration—the borrowing constraint shock and the house price beliefs shock—the monetary authority faces a trade-off between stabilizing borrowers' consumption or house prices, which is equivalent to stabilizing savers' consumption. This trade-off persists despite the fact that the shocks originate in different sectors of the economy: the borrowing constraint shock primarily impacts borrowers, while the house price beliefs shock is rooted in the savers' side of the economy.

Finally, we examine the case of inactive monetary policy, which can be viewed as approximating a scenario where the zero lower bound (ZLB) is binding. In response to a borrowing constraint shock, we find that house prices remain unaffected, while borrowers' consumption initially declines, then increases, before eventually returning to the steady state. In contrast, when subjected to a house price belief shock, both house prices and borrowers' consumption experience a simultaneous decline upon the shock's impact.

We next extend our model to incorporate precautionary savings motives by allowing for transitions between borrower and saver types, which occur with a certain exogenous probability. This modification alters the model's dynamics in one key dimension: savers become directly exposed to the borrowing constraint shock. This exposure arises because savers anticipate the probability of transitioning to borrower status in the next period, making them susceptible to future borrowing constraints. As a result, a shock to the borrowing constraint can simultaneously place downward pressure on house prices and reduce borrowers' consumption. With respect to the previously discussed channels,

[2]In our baseline model, prices are fully sticky, housing trade between agents is hut down outside of the steady-state,and precautionary motives are not present. In this setting, saver consumption is equivalent to the house price, which allows us to present the full model in two equations.

all mechanisms remain intact: the direct and indirect channels linking house prices to borrowers are active, house price extrapolation continues to amplify price volatility, and monetary policy affects the economy both contemporaneously and with a lag.

As a second extension, we allow households to engage in housing transactions outside of the steady-state, introducing a fire-sale motive. When a borrower faces a tightening of their borrowing constraint, they must cut spending, which can occur through either a reduction in consumption or the sale of housing. Depending on the parameterization of the model, both scenarios may be possible. If the borrower opts to sell housing, an equilibrium price is established at which the borrower sells while the counter-party, a saver, purchases. This situation can be described as a fire sale, as the borrower is selling at a price at which they would choose to buy if they were unconstrained, i.e. if they were a saver. In this context, house price extrapolation directly influences decision-making. Observed declines in house prices will be extrapolated into the future, incentivizing borrowers to sell housing sooner rather than later. In equilibrium, this increased housing supply will lead to further declines in house prices. Crucially, the sale of housing and the associated decrease in prices will tighten borrowing constraints even further, potentially triggering a downward spiral in both house prices and aggregate economic activity. This framework also highlights that the two shocks we consider—borrowing constraint shocks and house price belief shocks—are more aligned in their effects on housing markets, as both lead to increased selling activity among borrowers.

Finally, we study how monetary policy should optimally behave in this type of situation. For simplicity, we return to our baseline exercise and assume that the planer holds rational expectations. We show that the social planner aims to stabilize a weighted average of saver and borrower consumption. As discussed above, stabilizing both simultaneously is not possible and the planer therefore faces a trade-off. We show that this trade-off is static under rational expectations and dynamic under extrapolation. The intuition is as follows: under extrapolation raising interest rates affects current house prices, but it also affects the future house price expectations, thereby introducing a forward looking component. The optimal policy of the planer is, in response to a contractionary house price belief or borrowing constraint shock, to first cut the interest rats and increase it slightly in the following period. The planer thereby implements a middle ground between full house price stabilization and full borrower consumption stabilization.

Literature review. Our paper contributes to a broad empirical literature that emphasizes the formation of house price beliefs deviating from the rational expectations framework.[3] This body of work identifies momentum and revisions in belief formation as critical elements in understanding house price dynamics. On the theoretical side, our study is linked to the behavioral macro-finance literature, which explores departures from rational expectations, particularly in the formation of asset price expectations.[4] More specifically, we align with the literature on capital gains extrapolation.[5] In the context of housing markets, Glaeser and Nathanson (2017) and Schmitt and Westerhoff (2019) model house price expectations using forms of extrapolation within partial equilibrium frameworks. In contrast, we adopt a general equilibrium New Keynesian framework, positioning our work closer to studies such as Adam et al. (2012), Caines and Winkler (2021), and Adam et al. (2022). Additionally, other relevant contributions, such as Burnside et al. (2016), Guren (2018), and Kaplan et al. (2020), explain house price behavior through mechanisms such as optimism and pessimism, concave demand curves faced by sellers, or exogenous shifts in house price beliefs.

We also relate to the literature on aggregate demand externalities. For instance, Eggertsson and Krugman (2012) study the effect of debt deleveraging when the zero lower bound binds and the implications for fiscal policy. Korinek and Simsek (2016) focus on macroprudential policy implications in a similar setup. Both papers, however, abstract from belief extrapolation. Our paper most closely relates to Fontanier (2022) and Farhi and Werning (2020). Both of these paper study aggregate demand externalities under asset price belief extrapolation and the consequences for monetary and macroprudential policies. In contrast to these models, our framework incorporates general equilibrium effects through income dynamics, allowing us to capture feedback mechanisms between house prices and broader economic variables. Furthermore, we emphasize the scenario in which agents actively trade the asset—housing—about which they form extrapolative expectations.

Outline. The rest of this paper is organized as follows. In Section (II) we introduce our general model framework. Section (III) outlines our baseline model and discusses the two primary channels that are operative within it. In Section (IV), we present two extensions to the baseline model, while Section (V)focuses on the analysis of optimal monetary policy. Finally, Section (VI) concludes.

[3]See, for example, Case et al. (2012); Armona et al. (2019); Kuchler and Zafar (2019); Ma (2020).

[4]Among others, see: Bordalo et al. (2018); Barberis (2018); Caballero and Simsek (2019, 2020); Krishnamurthy and Li (2020); L'Huillier et al. (2023); Maxted (2024); Bianchi et al. (2024).

[5]Adam et al. (2017) and Winkler (2020) investigate asset price learning in stock markets.

II. General Model Setup

In this section, we describe the model setup. It draws largely from Bilbiie (2024). The household side consists of a borrower and a saver. The borrower faces a borrowing constraint, which depends on house prices and always binds. The saver is a standard unconstrained household. Firms are monopolistic and competitive and face price adjustment costs, as is standard in the New Keynesian literature. Finally, housing supply ($\bar{H}$) is assumed to be fixed.

Households. The household block consists of a borrower (h), or HtM, and a saver (s). Borrowers are more impatient than savers, $\beta^s > \beta^h$, and as a result borrowers will always be on the borrowing constraint which we define below. Households maximize utility choosing consumption c^i, hours worked n^i, housing h^i, and bonds b^i. Within the group of savers/borrowers, there is perfect insurance, hence all households within a group make the same decisions. Households stay with probability $p(i) = h/s$ borrower/saver. If this probability is one for both groups, we are in a classical TANK economy. When agents transition between types, they retain their liquid assets, consistent with the approach outlined in Bilbiie (2024). As housing is considered a illiquid asset, this does not hold for housing. Households are required to relinquish their housing assets without receiving any compensation when switching types. Thereby the housing stock remains with their original group. We also assume that each typ of household is endowed with an initial housing stock of H^i.

As is standard in the literature on capital gain extrapolation (e.g. Adam and Marcet, 2011; Adam et al., 2017), households are endowed with a set of beliefs in the form of a probability measure over the full sequence of variables that they take as given, henceforth external variables. This measure we denote as $\mathcal{P}$. Rational expectations are a special case of this setup in the form that households' beliefs agree with the objective, or equivalently "true" or "equilibrium-implied", distribution of external variables, $\mathcal{P} = \mathbb{P}$. Household hold rational expectations with respect to all variables but house prices. Below we will be more precise on the house price belief formation process. The utility function is denoted in the following way:

$$\max_{(c_t^i, n_t^i, h_t^i, b_{t+1}^i)_{t\geq 0}} \mathbb{E}_0^{\mathcal{P}} \sum_{t=0}^{\infty} (\beta^i)^t U(c_t^i, n_t^i, h_t^i), \quad U(c_t^i, n_t^i, h_t^i) = \frac{\xi_c^i (c_t^i)^{1-\sigma}}{1-\sigma} + \frac{\xi_h^i (h_t^i)^{1-\nu}}{1-\nu} - \frac{\xi_n^i (n_t^i)^{1+\varphi}}{1+\varphi} \tag{1}$$

Households consume, borrow or lend in bonds, buy housing, which is subject to a quadratic adjustment cost, receive/pay interest on bonds, receive income, and finally receive taxes and transfers. The budget constraint reads:

$$c_t^i + b_{t+1}^i + q_t[h_t^i - (1-\delta^i)h_{t-1}^i] + \kappa_H^i(h_t^i - h_{t-1}^i)^2 = \\ (1+r_t)[p(i)b_t^i + (1-p(-i))\frac{d(-i)}{d(i)}b_t^{-i}] + w_t n_t^i + \Sigma_t^i + T_t^i \quad (2)$$

$d(i)$ is the share of a certain household type in economy, $d(h) = \lambda$ denotes the share of borrowers, and $d(s) = 1 - \lambda$ the share of savers. We make the same assumption regarding profits and taxes as Bilbiie (2024). Only savers receive profits, which are then taxed and redistributed to borrowers. The taxation schedule is chosen such that counter-cyclical income risk arises, which is the empirically plausible case. Households are subject to a borrowing constraint which is given by:

$$b_{t+1}^i \leq q_t h_t^i \phi_t \quad (3)$$

ϕ_t is exogenous and can be thought of as a shock to the borrowing constraint.

House price beliefs. We follow Fontanier (2022), and define house price belief formation as:

$$\mathbb{E}_t^{\mathcal{P}} q_{t+1} = \mathbb{E}_t q_{t+1} + \alpha(q_t - q_{t-1}) + \epsilon_t^q \quad (4)$$

The log-linearized formulation therefore is given by:

$$\mathbb{E}_t^{\mathcal{P}} \widehat{q}_{t+1} = \mathbb{E}_t \widehat{q}_{t+1} + \alpha(\widehat{q}_t - \widehat{q}_{t-1}) + \widehat{\epsilon}_t^q \quad (5)$$

Variables denoted by "$\widehat{\ }$" represent percent deviations from the steady-state. As the belief shock, ϵ_t^q, is zero in steady-state, we define the deviations from steady-state relative to the steady-state house price, $\widehat{\epsilon}_t^q = \frac{\epsilon_t^q}{q_{ss}}$. The parameter $\alpha \in [0, 1)$ indicates the degree of extrapolation, reflecting the assumption that households anticipate future house prices, or equivalently house price growth, based on past observed growth. This formulation includes the case of rational expectations as a special case,$\alpha = 0$. Thus, extrapolation, as captured by this model, serves as a shifter around the rational expectations benchmark.

While this approach may seem somewhat ad hoc, it effectively captures the essential feature of extrapolation while remaining sufficiently parsimonious to permit analytical solutions. An alternative, more empirically grounded formulation is proposed by Adam et al. (2022), which similarly models belief updates as a function of past house price growth.

Firms and price setting. We assume a continuum of monopolistically competitive firms that produce intermediate good varieties and have the same beliefs as households. Firm beliefs, however, concern only variables over which households have rational expectations. Therefore, firms are rational. Firm j buys labor $n_t(j)$ from the representative labor packer and produces the variety $y_t(j)$ with a linear technology where labor is the only production factor. The firm sets its retail price $P_t(j)$ and maximizes the expected discounted stream of profits, subject to Rotemberg-type adjustment costs. The log linearized Phillips-Curve is given by:

$$\widehat{\pi}_t = \beta \mathbb{E}_t \widehat{\pi}_{t+1} + \frac{\epsilon - 1}{\kappa} \widehat{w}_t \tag{6}$$

Steady-state, Market clearing and Equilibrium. To solve the model we take a first-order approximation around the non-stochastic steady-state. This steady state is equivalent for the rational expectations and the extrapolation model. We ensure that the steady-state is efficient, by including firm subsidy as is standard in the literature. We further choose the steady-state preference shifters $\xi^i_{c,h}$ and define a steady-state lump sum redistribution tax such that steady-state consumption, housing, and labor choices are equated across household groups in steady-state.[6]

For our baseline model we will assume that $\kappa^s_H = 0$ and $\kappa^h_H \to \infty$. Further, we assume $\delta^h = 0$. It follows, given a fixed housing stock, that households simply hold their housing endowments H^i in steady-state.

In equilibrium labor, goods, and housing markets need to clear. Further, the monetary authority sets the nominal interest rate, i_t, according to a rule that remains unspecified at this point.

Definition 1 (Internally Rational Expectations Equilibrium)**.** *An IREE consists of three bounded stochastic processes: shocks $(\phi_t, \epsilon^q_t)_{t\geq 0}$, allocations $(c^i_t, b^i_t, h^i_t, n^i_t)_{i=s,h}$ and prices $(w_t, q_t, i_t, [P_t(j)]_{j\in[0,1]}$, such that in all t*

1. *households choose $(c^i_t, b^i_t, h^i_t, n^i_t)_{i=s,h}$ optimally, given their beliefs $\mathcal{P}$,*
2. *firms choose $([P_t(j)]_{j\in[0,1]}$ optimally, given their beliefs $\mathcal{P}$,*
3. *the monetary authority acts according to a certain rule,*
4. *markets for consumption good varieties, hours, and housing clear given the prices.*

[6] In steady-state we have that $c_{ss} = c^h_{ss} = c^s_{ss} = y_{ss} = n_{ss} = n^h_{ss} = n^s_{ss}$. For simplicity we set $y_{ss} = 1$ and choose housing supply $\bar{H}$ & ξ^i_h such that $q_{ss}\bar{H}\bar{\phi} = 1$.

III. Baseline model

We start by characterizing our baseline model. We make simplifying assumptions that will allow us to derive analytical results and boil down the interactions between borrower consumption and house prices to an indirect, general equilibrium effect, and a direct effect working through borrowing constraints. We view this as the most basic interaction between house prices and borrower consumption. Throughout this section, we will assume that prices are fully sticky, $\kappa \to \infty$. The central bank therefore directly sets the real interest rate and we can ignore the Phillips-Curve.

III.A Model solution

We first assume that $\kappa_H^s = 0$ and $\kappa_H^h \to \infty$. It follows, given a fixed housing stock, that no trade in housing between household groups takes place. Further, we assume $\delta^h = 0$. Under these assumptions, borrowers will make no adjustments to their housing stock and will always hold the same amount of housing. Consequently, housing drops out from their budget constraint, and their housing demand is non-essential. Additionally, we will consider a TANK economy, $s = h = 1$. This assumption shuts down precautionary savings motives. For notational ease set $\sigma = 1, \varphi = 1, \xi_n^{h,s} = \frac{1}{2}$. For the savers, the Euler equation, the housing demand equation, and the FOC wrt. housing is given by:

$$\widehat{c}_t^s = \mathbb{E}_t^{\mathcal{P}} \widehat{c}_{t+1}^s - \widehat{r}_{t+1} \tag{7}$$

$$\nu \widehat{h}_t^s = \frac{1}{(1-\bar{\beta})}(\widehat{c}_t^s - \bar{\beta}\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+1}^s) - \frac{\widehat{q}_t - \bar{\beta}^s \mathbb{E}_t^{\mathcal{P}} \widehat{q}_{t+1}}{1-\bar{\beta}^s} \tag{8}$$

With $\bar{\beta}^s = (1-\delta)\beta^s$. In our model households make choices given their beliefs about variables perceived to be external to them. These variables consist of prices and shocks. However, not all beliefs about external variables follow rational expectations. In our case house price expectations are formed according to equation (5). All other prices are formed according to rational expectations. For these variables the standard expectation formation applies and we can drop the $\mathcal{P}$ from the expectations operator. It remains to characterize expectations of variables that are internal to the household. In our model this is future expected consumption of the saver ($\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+1}^s$). As these future household choices depend on future prices the household holds non-rational expectations about, i.e. house prices, we have to characterize these variables under their subjective expectations, or henceforth subjective expectations.

In principle, it is possible to characterize future choices under subjective expectations. This is however tedious and closed form characterizations are rarely possible. We therefore assume housing utility to be linear, $\nu = 0$. This is a crucial assumption as housing drops as a state variable from the model. This allows us to cast the model in its most basic form.

Lemma 1 (Model solution)**.** *Under the assumptions stated in the main text savers consumption is given by:*

$$\widehat{c}_t^s = \widehat{q}_t$$

It therefore follows that:

$$\mathbb{E}_t^{\mathcal{P}} \widehat{c}_{t+1}^s = \mathbb{E}_t^{\mathcal{P}} \widehat{q}_{t+1}$$

Proof. Iterating on the housing demand equation gives the result. ■

Lemma (1) states that the savers' consumption choices follow the house price, and therefore the subjective expectations of the savers' consumption must also follow the expected house price path. We can therefore substitute out savers' consumption from the model, which allows us to fully characterize the model in two equations.

Proposition 1 (Baseline model)**.** *Under the given assumption the model can be reduced to the following two equations*

$$\widehat{q}_t = \mathbb{E}_t \widehat{q}_{t+1} + \alpha(\widehat{q}_t - \widehat{q}_{t-1}) + \widehat{\epsilon}_t^q - \widehat{r}_{t+1} \tag{9}$$

$$(1 - \lambda\chi_1)\widehat{c}_t^h = \Big[(1-\lambda)\chi_1 + \chi_2\Big]\widehat{q}_t + \chi_2\widehat{\phi}_t - \chi_3\Big[\widehat{q}_{t-1} + \widehat{\phi}_{t-1} + \widehat{r}_t\Big] \tag{10}$$

With $\chi_1 > 1, \chi_2 \in (0,1), \chi_3 \in (0,1)$.

Proof. See Appendix (A). ■

The final two equations consist of two endogenous choice variables, borrower consumption ($\widehat{c}_t^h$), and house prices or savers consumption ($\widehat{q}_t$). Further, there is one policy variable ($\widehat{r}_{t+1}$), and two the shocks (ϵ_t^q, ϕ_t).

Equation (9) pins down house prices or savers consumption and can be thought of as a standard IS-equation. Three properties are noticeable. First, it becomes clear that one channel through which monetary policy works is by determining asset/housing prices. In period t the monetary authority sets the interest rate $\widehat{r}_{t+1}$ and thereby influences house prices. Through this channel the monetary authority can *contemporaneously* affect the economy. Second, extrapolation enters the model through this equation. Under the assumptions made above, savers are the only agents for which expectations matter, thereby the IS-equation constitutes the only channel through which extrapolation matters. Third, as house price expectations only show up in the IS-equation, house price expectations shocks are also only situated in this equation.

Equation (10) pins down borrower consumption. It is determined by three parts: First, current house prices ($\widehat{q}_t$), which can be decomposed into a direct and an indirect effect as discussed below. Second, the current shock to the borrowing constraint ($\widehat{\phi}_t$). Third, the repayment of last period's debt. This is a function of the past borrowing constraint shock ($\widehat{\phi}_{t-1}$) and the past house price ($\widehat{q}_{t-1}$). Crucially, this is the second channel through which monetary policy affects the economy. As the central bank chooses the interest rate, it will also determine the next period's repayment schedule by borrowers. Monetary policy thereby affects the economy with a *lag*.

Finally, one should note that the two shocks we consider here, a shock to house price beliefs, and the borrowing constraint shock, enter the model through different equations. Thereby, this model formulation uncovers a fundamental difference between both shocks: one works through savers' consumption-savings choice, while the other affects borrowers' consumption through the ability to borrow. As shocks differ, policy responses to the shocks may also differ. We will return to this below.

III.B Channels

We will now turn to the specific channels that drive the interaction between house price extrapolation and consumption of the borrowers.

Amplification through extrapolation. As stated above, in our baseline model house price extrapolation enters only through the IS-equation. Therefore, any interaction between house price extrapolation and borrowers' consumption needs to pass through house prices. We start by characterizing the effect of extrapolation on house prices.

Lemma 2 (Pass-through from beliefs to prices)**.** *Consider an economy where $\widehat{\epsilon}_t^q = 0, \forall t$. Suppose the monetary authority changes $\widehat{r}_{t+1}$ in period t and is inactive thereafter. The house price in t is given by:*

$$\widehat{q}_t = \frac{-\widehat{r}_{t+1}}{1-\alpha}$$

The house price in t is increasing in the degree of extrapolation α.

Proof. Follows immediately from equation (9) and the fact that $\mathbb{E}_t\widehat{q}_{t+s} = 0 \forall s > 1$ since monetary policy is only active in t. ■

Lemma (2) transparently shows, that stronger extrapolation leads to a stronger response in house prices on impact. Or equivalently, monetary policy becomes more potent as the degree of extrapolation rises. Also, note that a similar statement can be made concerning the house price beliefs shock.

Pass-through to borrower consumption. After establishing that extrapolation leads to an amplification in the responsiveness of house prices, we now turn to the effect of house price variation on borrower consumption. Making use of the goods market clearing condition, $\widehat{y}_t = \lambda \widehat{c}_t^h + (1-\lambda)\widehat{q}_t$, we can make the following statement.

Lemma 3 (borrower consumption: direct & indirect effect)**.** *The borrower consumption response to current house prices, $\widehat{q}_t$, can be decomposed into a direct and an indirect effect:*

$$\widehat{c}_t^h = \underbrace{\chi_1 \widehat{y}_t}_{indirect} + \underbrace{\chi_2 \widehat{q}_t}_{direct} + \chi_2 \widehat{\phi}_t - \chi_3 \left[\widehat{q}_{t-1} + \widehat{\phi}_{t-1} + \widehat{r}_t \right]$$

Proof. Follows immediately from equation (10) and goods markets clearing. ∎

Lemma (3) decomposes the pass-through from current house price changes to borrowers' consumption into two effects. First, an indirect effect that captures the response of output to house prices. The intuition is as follows: house prices change savers consumption, which affects aggregate demand and wages. Eventually, this passes through to borrower consumption. This effect is a typical general equilibrium effect which is present in many HANK models as discussed in Bilbiie (2024).

Second, a direct effect of house prices on the ability to borrow. This effect is quite straightforward: house prices tighten or loosen the borrowing constraint, which is passed on to borrower consumption. Both channels are affected by extrapolation only indirectly through house prices.

III.C Dynamic propagation of shocks

We will now turn to our two shocks and study their propagation through the model. Proposition (1) shows that there are only two endogenous choice variables and one policy variable in the model. We will therefore consider the cases when monetary policy, in response to one of the two shocks, chooses to stabilize one of the two choice variables. Specifically, monetary policy will either stabilize house prices (saver consumption), or borrower consumption. Any other sensible policy will lie between these two extreme cases. This is also the case for optimal policy, as shown below. In all the cases we will simply consider a one-time drop in the borrowing constraint ($\widehat{\phi}_t$), or expected future house prices ($\widehat{\epsilon}^q_t$).

For the following expositions we set φ and σ to one. Further, we assume that 25% of the population are borrowers, hence $\lambda = 0.25$. We choose a tax rate of $\tau^d = 0.25$ which assures counter-cyclical income risk. Finally, for the extrapolation case we set $\alpha = 0.5$.

Borrowing constraint shock. Equation (10) illustrates, that to stabilize borrower consumption the central bank needs to raise house prices. This in turn requires to lower rates in the period the shock hits. Consequently, it is impossible for the monetary authority to stabilize both, house prices and borrower consumption, at the same time. Stabilizing house prices on the other hand requires for the central bank to stay inactive. This in turn leads to movements in borrower consumption.

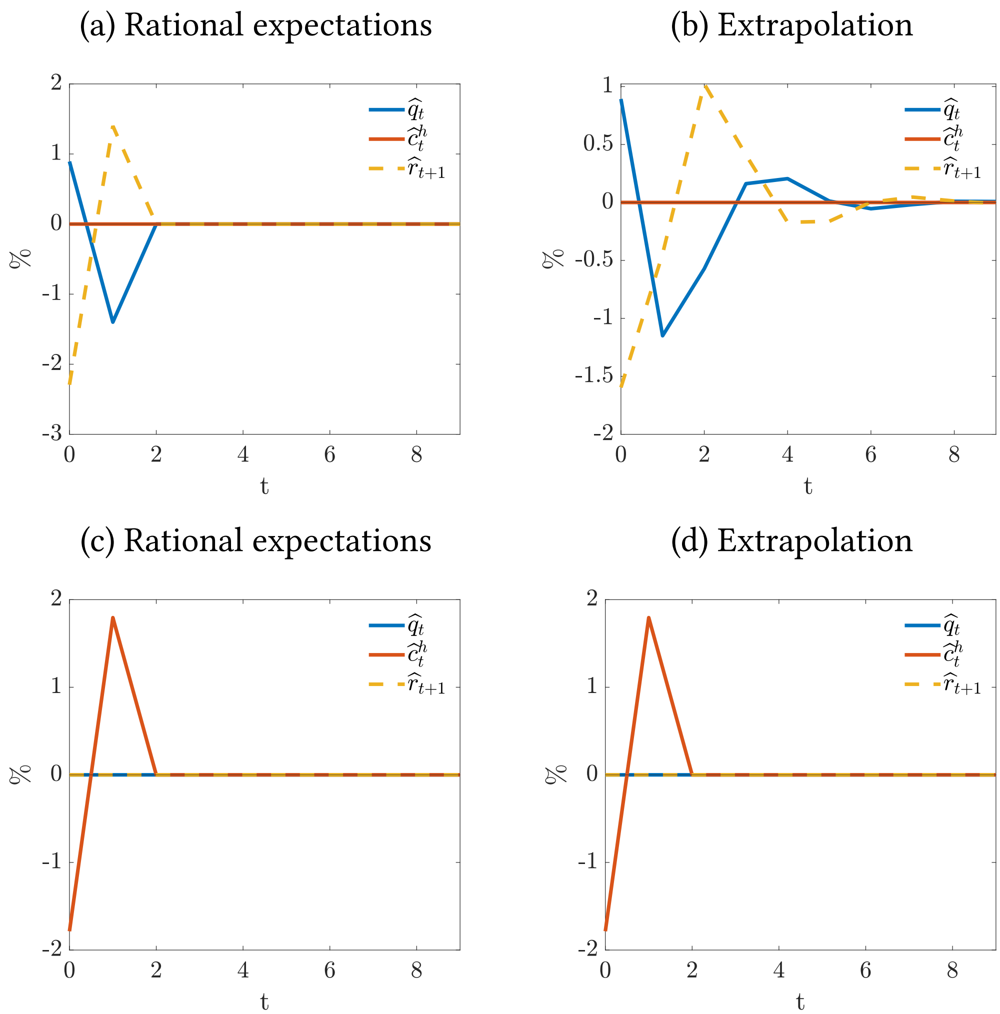
Figure (1) shows the cases when monetary policy stabilizes borrower consumption, under rational expectations and extrapolation, panel (a) and (b) respectively. Focusing on the rational expectations case first, to stabilize borrower consumption on impact the central bank raises house prices by cutting rates. Due to the backward-looking nature of the borrower budget constraint, the central bank needs to raise rates in the subsequent period to prevent a boom in borrower consumption. The reason is that due to lower rates in the first period, borrowers need to pay back less which puts upward pressure on consumption. From period three onwards, the central bank can implement a policy such that the economy returns to the steady-state.

Under extrapolation, the logic is similar, only now a move in house prices today shifts expectations away from rational expectations due to extrapolation. The backward-looking nature of expectations leads to a persistence in house price expectations, and thereby in house prices. The monetary policy authority can only lead the economy back to the steady-state after several periods.

Finally, one should note that the central bank needs to move rates by less under extrapolation to respond to the shock. This reflects the increased potency of monetary policy as stated in Lemma (2).

Next, we focus on the case when the central bank stabilizes house prices. As already pointed out above, house prices are not directly affected by the borrowing constraint shock, and respond only to interest movements. Therefore, the monetary authority remains inactive. This leads to a decrease of borrower consumption on impact, and, since households borrowed less in the last period, to a boom in the subsequent period (panel (c)). Since house prices do not move, there is also no difference between a rational expectations model and an extrapolation model (panel (d)).

Figure 1: IRFs to a one-time shock to the borrowing constraint



Notes: The figure shows IRFs of house prices, borrower consumption, and the interest rate to a borrowing constraint shock. Panel (a) and (b) consider the case of stabilizing borrower consumption, while panel (c) and (d) consider the case of house price stabilization.

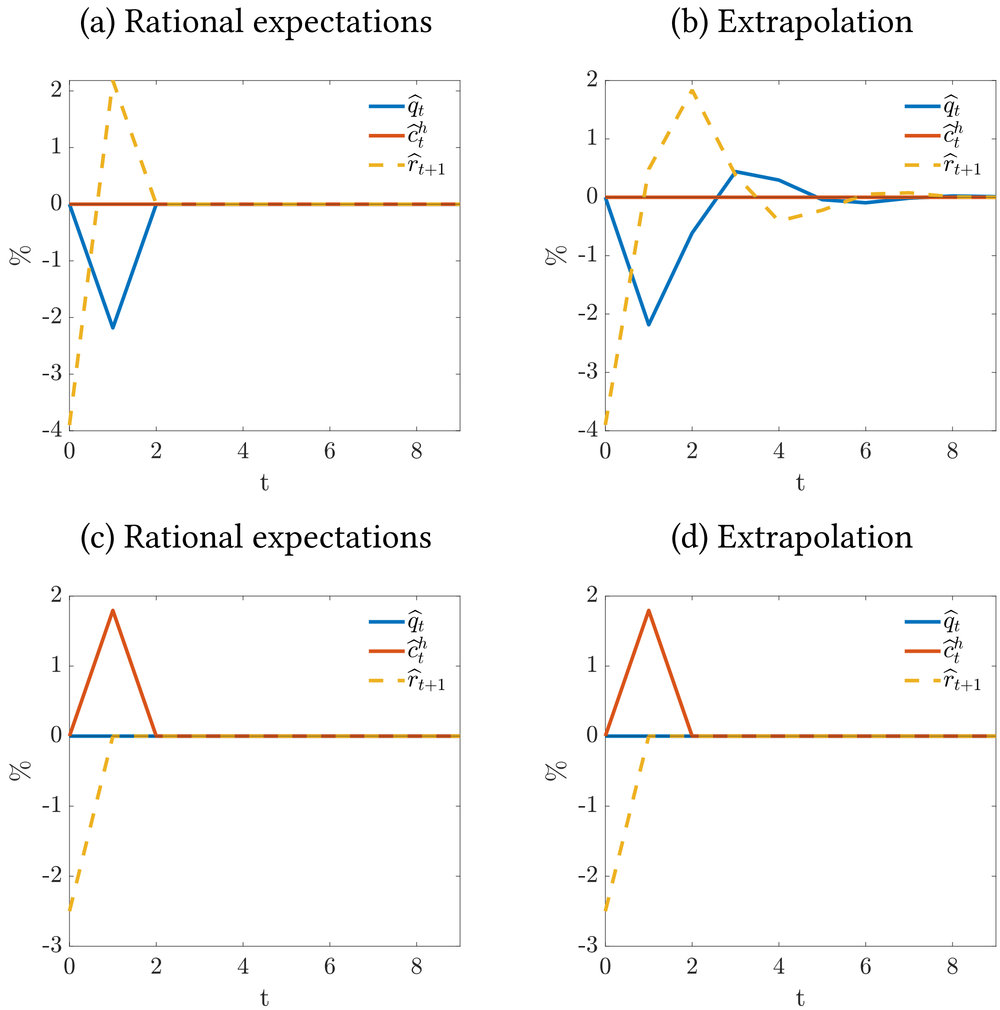
House price expectations shock. We will now focus on the house price expectations shock. This shock is situated in the IS-equation (9) and it is therefore possible to stabilize borrower consumption *and* house prices in the period the shock hits. This holds for rational expectations and extrapolation. One could interpret this result as a type of divine coincidence which lasts however only for one period. Due to the backward-looking nature of the borrowers' budget constraint, full stabilization fails in the subsequent period.

Figure (2) presents a numerical illustration.

Under rational expectations and borrower consumption stabilization, the central bank needs to raise rates to prevent an increase in borrowers' consumption. Consequently, house prices fall as shown in panel (a). After two periods the economy returns to the steady-state. Under extrapolation the dynamic is again more persistent for the same reasons as discussed above, see panel (b).

In case the central bank chooses to stabilize house prices, it cuts rates on impact and remains inactive thereafter. The lower interest rates from the impact period push up borrower consumption in the subsequent period, after which the economy returns to the steady-state. Again, as house prices do not move, there is no difference between the rational expectations case and the extrapolation case, see panel (c) and (d).

Figure 2: IRFs to a one-time shock to house price expectations



Notes: The figure shows IRFs of house prices, borrower consumption, and the interest rate to a house price belief shock. Panel (a) and (b) consider the case of stabilizing borrower consumption, while panel (c) and (d) consider the case of house price stabilization.

III.D Inactive monetary policy

Financial crises represent extreme events that place significant stress on economic systems. In such scenarios, the monetary authority may find itself constrained, particularly when policy rates approach the zero lower bound (ZLB). If this is the case aggregate demand externalises arise further amplifying the crises. In this section, we examine the dynamics of shock propagation within the economy under conditions where monetary policy becomes inactive. For the sake of analytical clarity, we assume that the monetary authority remains passive following the initial impact of the shock. However, in practice, if the shock is sufficiently large, the central bank might reduce interest rates until the ZLB is reached, at which point policy becomes constrained. This situation can be viewed as a combination of the transmission mechanisms under active monetary policy, previously discussed, and those under inactive monetary policy, which we will be focusing on now.[7] As before, we will analyze the implications of inactive monetary policy in the context of both a borrowing constraint shock and a house price expectations shock.

Borrowing constraint shock. In the event of a borrowing constraint shock, the resulting dynamics are relatively straightforward. The shock manifests within the borrower's budget constraint, as outlined in equation (10). When monetary policy is inactive, house prices remain unaffected, since monetary policy influences savers' decisions through the Euler equation (9). As a result, the situation leads to a scenario where monetary policy effectively "stabilizes" house prices but is unable to stabilize borrower consumption, as depicted in Figure (1). Hence, borrowers' consumption declines on impact and increases in the subsequent. Since house prices are unaffected extrapolation does not play any role.

House price expectations shock. In contrast, the house price expectations shock occurs in the Euler equation (9). To simplify our analysis we will assume for the following that the central bank is only inactive on the impact of the shock and thereafter ensures that $\mathbb{E}_t^{\mathcal{P}} \widehat{q}_{t+j} = 0, \forall j \geq 1$. We therefore have that in period t the house price is given by:

$$\widehat{q}_t = \frac{1}{1-\alpha} \widehat{\varepsilon}_t^q$$

We see that the house price decreases on impact and the contraction in house prices is increasing in the degree of extrapolation. As a result, borrowers' consumption also needs to decline: house prices depress aggregate activity and tighten the borrowing constraint and thereby lead to a decrease in borrower consumption as stated by Lemma (3). In summary, in response to a house price expectations shock, both house prices and borrower consumption decline, while extrapolation amplifies this dynamic.

Discussion. During the Great Recession, a significant contraction was observed in both asset prices and borrower consumption. Our model is capable of replicating this

[7]This scenario may also be interpreted as the central bank being constrained in a steady-state environment.

dynamic under inactive monetary policy—such as that seen during the Recession—and contingent upon a house price expectations shock. However, this outcome does not align with empirical evidence. Specifically, Mian and Sufi (2015) provide compelling evidence that the Great Financial Crisis was driven by a shock to the borrowing constraint, which contrasts with the results generated by our model. This discrepancy suggests that our theoretical framework is inaccurate. The fundamental issue lies in the model's oversimplification. While it distills the core dynamics, it overlooks critical channels that become particularly relevant during periods of financial crisis. In the subsequent sections, we will relax several of the model's assumptions, demonstrating that this leads to an improvement in its performance. This approach offers the advantage of transparently decomposing the various transmission channels at play.

IV. Extensions

In this section we will consider two extensions to our model. First, we will allow for a precautionary savings motive by allowing households to switch types. Second, we will allow for housing trade to take place. This extension introduces fire sale motives into the model.

IV.A Precautionary savings

To allow for precautionary savings motives we assume that $s, h \in (0,1)$. In Bilbiies' language, we switch from a TANK economy to a THANK economy. Under this assumption the Euler equation (7) is replaced by a THANK type Euler equation:

$$\widehat{c}_t^s = s\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+1}^s + (1-s)\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+1}^h - \widehat{r}_{t+1} \tag{11}$$

A positive probability of switching from a saver to a borrower, $s \in (0,1)$, introduces a precautionary savings motive through $(1-s)\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+1}^h$ in the Euler equation: if borrower consumption is expected to decline in the future, the saver reduces consumption and increases savings. Applying Lemma (1) and making use of the fact that borrower consumption is given by equation (10) we can make the following statement:

Proposition 2 (Precautionary savings). *Under the given assumption the model can be reduced to the following two equations*

$$\widehat{q}_t = s\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+1} + (1-s)\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+1}^h - \widehat{r}_{t+1} \tag{12}$$

$$(1-\lambda\chi_1)\widehat{c}_t^h = \Big[(1-\lambda)\chi_1 + \chi_2\Big]\widehat{q}_t + \chi_2\widehat{\phi}_t - \chi_3\Big[\widehat{q}_{t-1} + \widehat{\phi}_{t-1} + \widehat{r}_t\Big] \tag{13}$$

$$\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+1}^h = \frac{1}{2}\Big[\omega_1\mathbb{E}_t\widehat{w}_{t+1} + \mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+1} + \mathbb{E}_t\widehat{\phi}_{t+1} - \omega_2(\widehat{q}_t + \widehat{\phi}_t + \widehat{r}_{t+1})\Big] \tag{14}$$

With $\chi_1 > 1$, $\chi_2 \in (0,1)$, $\chi_3 \in (0,1)$, $\omega_1 = (2 - \frac{\tau^D}{\lambda})$ *and* $\omega_2 = (\beta^h)^{-1}(h - (1-s)\frac{1-\lambda}{\lambda})$.

Proof. See Appendix (B). ■

First, it is important to note that the budget constraint of borrowers remains largely unaffected, as only the definitions of χ_3 changes slightly. Second, the Euler equation of the saver, equation (12), now includes current and future realizations of the borrowing constraint shock due to the presence of future expected borrowers consumption ($\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+1}^h$).

But how does the presence of the precautionary savings motive affect the model dynamics with respect to the two shocks? The model dynamics with respect to the house price expectations shock remain qualitatively unchanged: only the pass-through from savers to borrowers is altered due to the precautionary savings motive on the saver site. Additionally, the backward looking component in the borrowers' budget constraint changes as χ_3 is affected by the precautionary savings motive. Generally, the quantitative differences are likely to be marginal as the probability of switching types is probably small and therefore s and h should be close to one.

With respect to the borrowing constraint shock the case is different. The shock appears in the Euler equation (12) in the current and the future period through the expectations about net periods borrowers consumption. The intuition is the following. In the event of an previously unanticipated shock in the current period, borrowers face liquidity constraints, leading to a reduction in borrowing. As a result, debt repayment obligations in the following period decrease. A saver, who may transition to a borrower in the subsequent period, will consequently face a lower debt burden. This expectation prompts savers to increase their consumption in the present period in anticipation of lower future liabilities. Conversely, if a shock is expected to occur in the following period, savers anticipate that their borrowing capacity may be constrained if they transition into borrowers. Consequently, they reduce their consumption in the current period to account for the potential limitations on future borrowing.

It turns out that this channel is essential in improving the shortcomings of the baseline model. Consider again the case with inactive monetary policy and a borrowing constraint shock that is persistent such that $\mathbb{E}_t\widehat{\phi}_{t+1} < \omega_2\widehat{\phi}_t$. And again assume that the central bank is only inactive on the impact of the shock and thereafter ensures that

$\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+j} = 0, \forall j \geq 1$. Under this assumptions house prices will fall due to the precautionary savings motive. Borrower consumption in the current period will fall because of the shock to the borrowing constraint and this will be further amplified due to the contraction in house prices. As a result, the model can generate a situation which *both* house prices and borrower consumption decline simultaneously. Precautionary savings motives therefore provide a possible explanation for the patterns observed in the Great Financial crises (Mian and Sufi, 2015). Key for this outcome to occur is a shock to the borrowing constraint that is persistent, or at least believed to be persistent.

IV.B Fire sales

We will now allow for housing trade to take place and set the housing adjustment costs to zero ($\kappa_H^s = \kappa_H^h = 0$). For simplicity we will return to our TANK set up and shut down the precautionary savings motives. The equilibrium on the housing market is determined by housing demand of the savers and the borrowers, as well as the housing market clearing:

$$\widehat{h}_t^s = \frac{1}{\bar{\nu}^s}\Big(\widehat{c}_t^s - \bar{\beta}^s\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+1}^s - \widehat{q}_t + \bar{\beta}^s\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+1}\Big) \tag{15}$$

$$\widehat{h}_t^h = \frac{1}{\bar{\nu}^h}\Big(\widehat{c}_t^h - \bar{\beta}^h\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+1}^h - \widehat{q}_t + \bar{\beta}^h\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+1}\Big) \tag{16}$$

$$(1-\lambda)H^s\widehat{h}_t^s + \lambda H^h\widehat{h}_t^h = 0 \tag{17}$$

with $\bar{\nu}^i = \nu(1-\bar{\beta}^i)$ and H^h, H^s are the steady-state housing held by borrowers and savers. The heterogeneity in $\bar{\nu}^i$ and $\bar{\beta}^i$ arises because of the heterogeneity in β^i. For simplicity, we shut this down by allowing for heterogeneity in the housing depreciation rate (δ^i) and chose them such that $\bar{\beta}^h = \bar{\beta}^s$. Saver consumption is pinned down by the TANK version of the Euler equation (7). Further, goods markets must clear: $\widehat{y}_t = \lambda\widehat{c}_t^h + (1-\lambda)\widehat{c}_t^s$. Finally, borrower consumption is pinned down by the budget constraint:

$$\widehat{c}_t^h = \chi_1\widehat{y}_t - \chi_2 H^h q_{ss}(\delta^h\widehat{q}_t + \widehat{h}_t^h - (1-\delta^h)\widehat{h}_{t-1}^h) + \chi_2(\widehat{q}_t + \widehat{\phi}_t + \widehat{h}_t^h) - \chi_3(\widehat{r}_t + \widehat{q}_{t-1} + \widehat{\phi}_{t-1} + \widehat{h}_{t-1}^h) \tag{18}$$

The difference between this model version and the versions considered above is, that borrowers if forced to adjust consumption can also choose to adjust housing instead. For simplicity consider again a negative shock to the borrowing constraint ($\widehat{\phi}_t$). As before deleveraging forces borrowers to cut their consumption. To alleviate the loss in consumption borrowers will now sell housing. The degree to which households sell housing to balance out the loss in consumption is governed by the b equation (16). We can decompose the equation into two parts as follows:

$$\widehat{h}_t^h = \frac{1}{\bar{\nu}^h}\Big(\underbrace{\widehat{c}_t^h - \widehat{q}_t}_{contemporary} + \bar{\beta}^h (\underbrace{\mathbb{E}_t^{\mathcal{P}} \widehat{q}_{t+1} - \mathbb{E}_t^{\mathcal{P}} \widehat{c}_{t+1}^h}_{expectations})\Big)$$

The housing demand function can be decomposed into a contemporary and an expectations part. The contemporary part consists of a choice variable, consumption, and an external variable, house prices. A decrease in consumption will trigger a reduction in housing demand. At the same time, a decrease in house prices, which is plausible in a crisis, will lead to an increase in demand. As housing becomes cheaper, households demand more of this good, stabilizing housing demand. On the expectations side, a decline in anticipated future house prices dampens housing demand, as the prospect of lower prices reduces the incentive to hold onto housing, thereby encouraging current sales. A decline in expected future consumption increases housing demand, as housing can serve as a vehicle for intertemporal consumption smoothing. Whether expected consumption is increasing or decreasing in response to a borrowing constraint shock is ambiguous ex-ante. On the one hand, deleveraging leads to a lower debt burden to be repaid next period and boosts expected consumption. On the other hand, a more persistent crisis will dampen expected future consumption and so does house price extrapolation.

This exposition reveals an important channel that was absent in the other model versions discussed above. House price extrapolation now has a direct effect on borrower consumption: through the expectations part, it governs the degree to which borrowers are willing to substitute consumption for housing.

The described mechanism can be characterized as a "fire sale" motive. In response to a tightening of the borrowing constraint, borrowers are compelled to reduce consumption and potentially liquidate housing assets. Should they opt to sell housing, they will encounter an equilibrium price at which savers, acting as counterparties, are willing to purchase. In this equilibrium, borrowers are inclined to sell at a price below what they would have been willing to sell for in an unconstrained scenario. A phenomenon where an asset is sold at a discounted price relative to its value in an alternative situation, such as the unconstrained case in our scenario, is commonly referred to as a fire sale.

Combining equations (15), (16), and (17) illustrates the interaction of borrowers and savers on the housing market:

$$\widehat{q}_t = \bar{\beta}\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+1} + \Omega^{-1}\left(\widehat{c}_t^s + \bar{\nu}\frac{\lambda H^h}{(1-\lambda)H^s}\widehat{c}_t^h\right) - \bar{\beta}\Omega^{-1}\left(\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+1}^s + \bar{\nu}\frac{\lambda H^h}{(1-\lambda)H^s}\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+1}^h\right) \tag{19}$$

with $\Omega = 1 + \bar{\nu}\frac{\lambda H^h}{(1-\lambda)H^s}$. We observe that an expected increase in house prices raises current house prices. Additionally, an increase in consumption leads to an increase in house prices as higher consumption demand fuels demand for housing. Larger expected future consumption reduces house prices. From this formulation it becomes apparent how a reduction in borrower consumption interacts with house price extrapolation. Consider a one time drop in borrower consumption at $t = 1$, keeping all other variables in equation (19) fixed. Plugging in the house price expectations equation, assuming the economy was in the steady-state in $t-1$ and rearranging gives:

$$\widehat{q}_t = \frac{1}{1-\alpha}\left[\bar{\beta}\mathbb{E}_t\widehat{q}_{t+1} + \Omega^{-1}\left(\widehat{c}_t^s + \bar{\nu}\frac{\lambda H^h}{(1-\lambda)H^s}\widehat{c}_t^h\right) - \bar{\beta}\Omega^{-1}\left(\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+1}^s + \bar{\nu}\frac{\lambda H^h}{(1-\lambda)H^s}\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+1}^h\right)\right]$$

A larger degree of extrapolation, a higher α, amplifies the effect of the cut in borrower consumption on the house price. In equilibrium, a decline in house prices generates a feedback effect on borrower consumption. As a result, borrowers are compelled to further reduce their consumption due to the direct and indirect mechanisms previously discussed. This dynamic, in turn, intensifies the downward pressure on house prices. Consequently, this channel has the potential to exert significant influence, amplifying economic contraction and potentially pushing the economy into a deep recession. How does monetary policy connect to these dynamics? Monetary policy governs savers consumption choices through the Euler equation (7). Reducing the interest rate can therefore effect the intertemporal consumption decision by increasing today's saver consumption, and thereby housing demand, stabilizing the housing market. Notably, an increase in saver consumption also operates through the indirect channel as it stabilizes output. Monetary policy is therefore quite powerfully as it impacts the economy through various channels. As a result, monetary policy being constraint by the zero lower bound can be associated with large costs. Similar to the first extension, this model can therefore generally generate a simultaneous contraction in house prices and borrower consumption in response to a borrowing constraint shock.

As discussed in Section (III) deriving closed form solutions for the full equilibrium dynamics are not possible due to the inability to characterize the expected consumption term in closed form. For the solution of this model one has therefore to rely on numerical tools.

V. Optimal monetary policy

In this section, we analyse how monetary policy should be optimally conducted in this economy. For simplicity we will focus on our baseline model illustrated in Section (III). We assume that the social planner maximizes welfare under rational expectations, while the agents in the economy potentially from expectations on house prices by extrapolating on observed realizations. A second-order approximation of the borrowers and savers utility function yields the following loss function:

$$\mathbb{W}_{-1} = \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \Big[-(1-\lambda)(\widehat{c}_t^s)^2 - \lambda(\widehat{c}_t^h)^2 + 4\lambda(1-\lambda)(1-2\lambda)\widehat{c}_t^s \widehat{c}_t^h + O(3) + \text{t.i.p.} \Big] \tag{20}$$

Making use of Lemma (1) we can exchange $\widehat{c}_t^s$ with $\widehat{q}_t$. The social planner than maximizes equation (20) subject to the constraints given by equation (9) and (10). The solution to the planers' objective is given by:

$$\Lambda_1 \widehat{q}_t - \Lambda_2 \widehat{c}_t^h - \alpha\Lambda_3 \mathbb{E}_t \widehat{q}_{t+1} + \alpha\Lambda_4 \mathbb{E}_t \widehat{c}_{t+1}^h + \alpha\beta\Lambda_3 \mathbb{E}_t \widehat{q}_{t+2} - \alpha\beta\Lambda_4 \mathbb{E}_t \widehat{c}_{t+2}^h = 0 \tag{21}$$

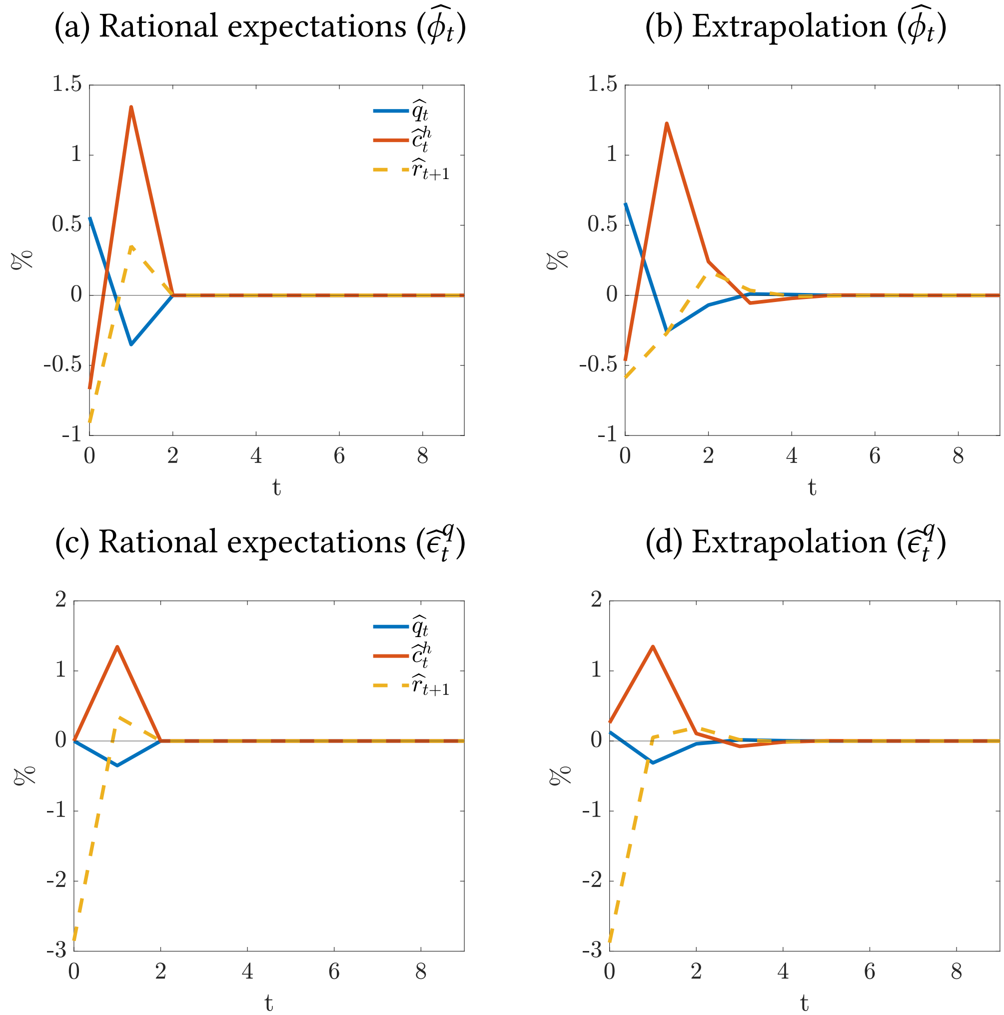
With $\Lambda_j > 0$. For the derivations and the definitions of the coefficients Λ_j the reader is referred to the Appendix (C). Equation (21) highlights two key points. First, the policymaker must balance borrower consumption against saver consumption. As demonstrated in Section (III), it is impossible to stabilize both house prices and borrower consumption simultaneously. Consequently, the optimal policy will be between the both extreme cases discussed above. It also shows that monetary policy on its' own is not sufficient to fully stabilize the economy. Second, under rational expectations ($\alpha = 0$), the policymaker faces a static trade-off between house prices and borrower consumption. However, with house price extrapolation, this trade-off becomes dynamic. The underlying intuition is that any policy action influencing current house prices will also affect house price expectations over the subsequent two periods, as realized house price growth shapes the formation of future expectations.[8]

Figure (3) presents a numerical illustration of the optimal monetary policy response

[8] Specifically, $\widehat{q}_t$ enters house price beliefs for periods $t+1$ and $t+2$. Therefore, the policymaker must account for the impact of $\widehat{q}_t$ on house price expectations over these successive periods.

to both a borrowing constraint shock and a house price expectations shock. In both cases, the shocks impact the economy only once, with their effects reduced to zero in subsequent periods. Panels (a) and (b) depict the optimal monetary policy response to a borrowing constraint shock. Under rational expectations, the policymaker is able to stabilize the economy within two periods. However, with house price extrapolation, the adjustment process takes longer, as previously discussed. Given that monetary policy is more effective under extrapolation, the optimal interest rate path is less reactive. Panels (c) and (d) illustrate the response to a house price expectations shock. Under rational expectations, the policymaker can stabilize the economy immediately upon impact, but due to the backward-looking nature of the borrower budget constraint, full stabilization is not achieved in the following period. Afterward, the economy returns to its steady state. Under extrapolation, the policymaker opts for a smoother adjustment, choosing not to fully stabilize the economy on impact. This is due to the fact that inducing a positive correlation between house price and borrower consumption is beneficial for welfare, as stated in equation (20). In the second period the economy experiences a boom in borrower consumption and a bust in house prices, after which the economy gradually returns to its' steady-state. Again, the potency of monetary policy is greater under extrapolation, allowing for a more moderated policy response.

Figure 3: Optimal monetary: borrowing constraint and house price belief shock



Notes: The figure shows IRFs of house prices, borrower consumption, and the interest rate to a borrowing constraint shock in panel (a), (b), and a house price belief shock in panel (c), (d).

VI. Conclusion

In this paper, we investigate the effects of house price belief extrapolation in an environment where housing serves as collateral for borrowers. We identify four key channels at work: a direct channel operating through borrowing constraints, an indirect channel arising from aggregate income fluctuations, a precautionary savings motive, and a fire sale channel. Lastly, we analyze the optimal conduct of monetary policy within this framework.

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Appendix

A. Proof Proposition (1)

Combining the labor supply equations of the saver and the borrower gives:

$$\varphi\widehat{n}_t^h + \sigma\widehat{c}_t^h = \varphi\widehat{n}_t^s + \sigma\widehat{c}_t^s \tag{A.1}$$

Further, the market clearing conditions for labor and consumption are given by:

$$\widehat{n}_t^s = \frac{1}{1-\lambda}(\widehat{y}_t - \lambda\widehat{n}_t^h) \tag{A.2}$$

$$\widehat{c}_t^s = \frac{1}{1-\lambda}(\widehat{y}_t - \lambda\widehat{c}_t^h) \tag{A.3}$$

Combining these equations gives labor supply of the borrower as function of output and borrower consumption:

$$\varphi\widehat{n}_t^h = (\sigma+\varphi)\widehat{y}_t - \sigma\widehat{c}_t^h$$

Given that $\kappa_H^h \to \infty$ and housing supply is fixed, we have that $\widehat{h}_t^s = \widehat{h}_t^h = 0$. The budget constraint of the borrower is then given by:

$$\widehat{c}_t^h = (1 - \frac{\tau^D}{\lambda})(\varphi\widehat{n}_t^h + \sigma\widehat{c}_t^h) + \widehat{n}_t^h + \widehat{q}_t + \widehat{\phi}_t - (\beta^h)^{-1}(h - (1-s)\frac{1-\lambda}{\lambda})(\widehat{q}_{t-1} + \widehat{\phi}_{t-1} + \widehat{r}_t)$$

Rearranging an substituting labor supply yields:

$$\widehat{c}_t^h = \underbrace{[1 + \varphi(1 - \frac{\tau^D}{\lambda})]}_{\chi_1}\widehat{y}_t + \underbrace{\frac{\varphi}{\varphi+\sigma}}_{\chi_2} + (\widehat{q}_t + \widehat{\phi}_t) - \underbrace{\frac{\varphi}{\varphi+\sigma}(\beta^h)^{-1}(h - (1-s)\frac{1-\lambda}{\lambda})}_{\chi_3}(\widehat{q}_{t-1} + \widehat{\phi}_{t-1} + \widehat{r}_t)$$

Note that in the TANK model we have that $s = h = 1$. Equation (10) can be derived by substituting output using the goods market clearing condition. Finally, equation (9) is derived by applying Lemma (1) to the Euler equation and substituting for the house price beliefs. This concludes the Proof.

B. Proof Proposition (2)

First, the budget constraint of the borrower can be derived as stated in Proposition (1). Only now we have that $s, h < 1$ and therefore χ_3 takes another value.

To derive equation (12) one first applies Lemma (1) to the THANK Euler equation (11). It now remains to characterize the borrowers' subjective expectations consumption in $t+1$, hence $\mathbb{E}_t^{\mathcal{P}} \tilde{c}_{t+1}^h$ in equation (11). To do this one can simply lead the borrowers budget constraint by one period. Substituting labor supply by the, first-order of the borrower condition with respect to labor supply gives the result. It is important to note, that we can not substitute out the expected wage using the market clearing conditions as the household is unable to understand that market clearing must hold in expectations. An expectations formulation of the expected wage can be derived using the savers' budget constraint. This concludes the proof.

C. Derivations for the optimal policy exercise

For simplicity we assume that $\sigma = \varphi = 1$ and refer to the TANK setup. The policy maker maximizes the following welfare objective by setting the real interest rate:

$$\mathbb{W}_{-1} = \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \Big[\lambda (ln c_t^h - \frac{1}{2}(n_t^h)^2) + (1-\lambda)(ln c_t^s - \frac{1}{2}(n_t^s)^2) \Big]$$

A second-order approximation around the steady-state, substitution of saver and borrower labor supply using labor and goods market clearing, and some final rearrangements yield equation (20).

The coefficients to the planners solution in equation (21) are given by:

$$\Lambda_1 = (1-\lambda) - 2\lambda(1-\lambda)(1-2\lambda)$$
$$\Lambda_2 = 2\lambda(1-\lambda)(1-2\lambda) + \lambda$$
$$\Lambda_3 = \frac{1}{1-\lambda\chi_1} 2\lambda(1-\lambda)(1-2\lambda)$$
$$\Lambda_4 = \frac{1}{1-\lambda\chi_1} \lambda$$