House Price Extrapolation and Business Cycles: A Two-Agent New Keynesian Approach

– preliminary and incomplete –

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Abstract

This paper examines the interaction between housing and business cycles in a tractable two-agent New Keynesian model featuring extrapolative house price beliefs. The model includes a saver and a hand-to-mouth borrower who uses housing as collateral. We identify four key transmission channels from housing markets to aggregate output: consumption, residential investment, collateral, and fire sales. Under rational expectations, output volatility is limited and primarily driven by consumption. In contrast, extrapolative beliefs significantly amplify output volatility, mainly through residential investment. Finally, we propose a novel solution approach for two-agent models with asset trade and asset price extrapolation, which is essential for solving models with fire sale motives.

JEL Codes: E4, E44, E52, E70

Keywords: Capital gains extrapolation, business cycles, heterogeneous agent models, monetary policy

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I. Introduction

Housing constitutes arguably the most important asset in household portfolios, functioning both as a durable consumption good and as a key form of collateral in credit markets. At the same time, house prices exhibit pronounced boom-bust cycles, often amplified by extrapolative over-optimism in expectations during expansions and over-pessimism during downturns, which contribute to substantial volatility in housing markets.[1] These characteristics position housing as a central component in the business cycle and underscore its importance for policy.

This paper offers two important contributions to the existing literature. First, we develop a tractable two-agent New Keynesian (TANK) model featuring a housing sector and extrapolative belief formation over house prices. This framework enables a decomposition of the transmission from changes in housing wealth to aggregate output through four distinct channels: consumption by savers (Ricardian households), housing investment, collateral constraints, and fire sale dynamics. Second, we introduce a novel solution method for TANK models with active asset trading between heterogeneous agents in the presence of belief-driven house price dynamics.

Our quantitative findings highlight that output responses are generally attenuated under rational expectations (RE) relative to subjective expectations (SE) characterized by extrapolative belief updating. Under RE, output dynamics are primarily driven by the consumption behavior of savers, whereas under SE, fluctuations in housing investment emerge as the dominant force. The collateral and fire sale channels, while theoretically relevant, contribute only marginally to the variation in aggregate output.

We begin our analysis by considering the representative agent New Keynesian (RANK) framework. Throughout the paper, we examine the effects of a contractionary monetary policy shock. In the RANK model, house prices respond to changes in the real rate through multiple channels: intertemporal consumption smoothing, housing supply dynamics, expectations over future house prices, and future expected wealth changes. This decomposition continues to hold under the extended TANK specifications. For the SE case, we assume internal rationality and solve the RANK version of the model as proposed by Roschitsch and Twieling (2024). Under SE, the house price response is substan-

[1] See e.g.: Case et al. (2012); Armona et al. (2019); Kuchler and Zafar (2019); Ma (2020); Kaplan et al. (2020).

tially amplified relative to the RE benchmark. This amplification is primarily driven by the more pronounced adjustment in house price beliefs. Aggregate output in the RANK model is shaped by two mechanisms: the consumption response of the representative (Ricardian) household and the dynamics of housing investment. We find that output is considerably more responsive under SE than under RE, a result largely attributable to the stronger housing investment response. While consumption is the dominant driver of output fluctuations under RE, housing investment becomes the primary transmission channel under SE.

Extending the analysis to a TANK framework, we incorporate non-Ricardian, or hand-to-mouth (HtM), households. These agents rely on housing as collateral to access credit, subject to an exogenous loan-to-value (LTV) constraint, which limits their borrowing capacity. In the baseline TANK specification, we assume that HtM households hold housing in the steady state but do not participate in housing market transactions outside of it. As a result, they are passive with respect to housing trade in response to shocks. This modeling choice enables us to isolate the collateral channel linking house prices to HtM consumption.

The output decomposition in this setting builds on the RANK model, now comprising three channels: savers' consumption, housing investment, and the collateral effect on HtM consumption. Consistent with previous findings, the output response to a monetary policy shock is significantly larger under SE than under RE. Under SE, housing investment remains the dominant driver of output fluctuations, whereas under RE, the primary contribution continues to stem from savers' consumption. The collateral channel has only a transitory influence on output, with limited quantitative differences between RE and SE regimes.

Finally, we turn to a version of the TANK model that allows for housing trade on the side of the HtM agents. In this model version, the HtM can sell or buy housing to stabilize consumption variations arising from exogenous shocks. Under SE the actual house price may not be aligned with the fundamental price due to extrapolative belief updating. It is therefore possible that agents sell housing at a price below the fundamental price to increase consumption, which is commonly referred to as a fire sale. A TANK model under SE and housing trade therefore allows for the emergence of fire sale motives.

However, this modeling approach introduces another layer of complexity. By allowing HtM to trade housing, we introduce an intertemporal choice to the HtM problem as housing is durable. As a result, we need to characterize beliefs about future HtM housing and consumption choices under SE. The method developed in Roschitsch and Twieling (2024) is not suitable for this task, as it relies on the consumption Euler equation, which is not binding for the HtM. We therefore propose to solve this problem by relying on a lag polynomial factorization, which allows us to characterize future expected household choices of the HtM as functions of prices.

Under RE, HtM households respond to a contractionary monetary policy shock by selling housing to stabilize consumption. Crucially, these transactions occur at prices consistent with fundamentals, as expectations are model-consistent. In contrast, under SE, HtM households initially increase housing in the quarters immediately following the shock. This behavior is driven by their expectations about future consumption. Given their high marginal propensities to consume (MPCs), anticipated declines in future income lead HtM households to forecast a corresponding drop in future consumption. To insure against this expected decline, they increase their current housing demand to smooth consumption. Due to the high MPCs, this effect is relatively strong in our model. After initially buying housing at overvalued prices, the HtM will sell housing in consecutive periods at undervalued prices, triggering fire sales.

In terms of aggregate output dynamics, the qualitative patterns are consistent with earlier model variants: output responses are more pronounced under SE relative to RE. Moreover, under SE, housing investment emerges as the primary driver of output fluctuations, whereas under RE, the consumption response of savers dominates. The collateral channel, while present, plays only a limited and transitory role in shaping the overall output response under both RE and SE. Finally, the fire sale channel is fairly transitory, but relative to the impact output response quite sizable.

Literature review. Leamer (2007) famously stated that "housing is the business cycle". Empirically, he documents that residential investment is an important driver of the business cycle. His empirical findings are in line with our findings under extrapolative house price beliefs, but at odds with the RE version of the model.

Our paper contributes to a broad empirical literature that emphasizes the formation of house price beliefs deviating from the rational expectations framework.[2] This body of work identifies momentum and revisions in belief formation as critical elements in understanding house price dynamics. On the theoretical side, our study is linked to the behavioral macro-finance literature, which explores departures from rational expecta-

[2]See, for example, Case et al. (2012); Armona et al. (2019); Kuchler and Zafar (2019); Ma (2020).

tions, particularly in the formation of asset price expectations.[3] More specifically, we align with the literature on capital gains extrapolation.[4]. In the context of housing markets, Glaeser and Nathanson (2017) and Schmitt and Westerhoff (2019) model house price expectations using forms of extrapolation within partial equilibrium frameworks. In contrast, we adopt a general equilibrium New Keynesian framework, positioning our work closer to studies such as Adam et al. (2012), Caines and Winkler (2021), and Adam et al. (2022). In addition to that, other relevant contributions, such as Burnside et al. (2016) and Guren (2018) explain house price behavior through mechanisms such as optimism and pessimism, or concave demand curves faced by sellers.

We also connect to the literature studying housing collateral effects. Mian et al. (2013) and Mian and Sufi (2015) document that during the great recession collateral constraints on housing were important drivers in explaining the decline in household consumption. In a theoretical context Guerrieri and Lorenzoni (2017) and Greenwald (2018) study the effects of collateral constraints on demand dynamics. Finally, Kaplan et al. (2020) develop a large-scale quantitative model that incorporates housing collateral constraints and deviations from rational expectations in house price beliefs to investigate the drivers of the Great Recession. Their analysis highlights the critical role of belief distortions—specifically, deviations from rational expectations—in accounting for the severity of the Great Financial Crisis. While sharing a focus on the macroeconomic implications of non-rational beliefs in housing markets, our approach differs in a key dimension: whereas Kaplan et al. (2020) impose exogenous belief processes over house prices, our model endogenizes the belief formation.

Outline. The rest of this paper is organized as follows. In Section (II) we introduce our general model framework. In Section (III), we present our results and the decompositions into the different channels. Finally, Section (IV) concludes.

II. General Model Setup

In this section, we outline the structure of the model. The heterogeneous agent framework builds on the approach developed in Bilbiie (2024). On the household side, the

[3] Among others, see: Bordalo et al. (2018); Barberis (2018); Caballero and Simsek (2019, 2020); Krishnamurthy and Li (2020); L'Huillier et al. (2023); Maxted (2024); Bianchi et al. (2024).

[4] Adam et al. (2017) and Winkler (2020) investigate asset price learning in stock markets.

economy comprises two types of agents: borrowers and savers. Borrowers are subject to a collateral constraint that is tied to the value of housing and binds at all times. In contrast, savers correspond to standard Ricardian households who are unconstrained. The production sector consists of monopolistically competitive firms operating under nominal rigidities in the form of price adjustment costs, consistent with the canonical New Keynesian framework

Households. The household block consists of a borrower (h), or HtM, and a saver (s). Borrowers are more impatient than savers, $\beta^s > \beta^h$, and as a result, borrowers will always be on the borrowing constraint, which we define below. Households maximize utility choosing consumption c^i, hours worked n^i, housing h^i, and bonds b^i. The savers can additionally invest into building new housing units, by committing x_t^s consumption units to it, hence ($x_t^h = 0, \forall t$). Within the group of savers/borrowers, there is perfect insurance, hence, all households within a group make the same decisions. Households stay within their group, and we therefore arrive in a standard TANK setting. $\mathbb{E}_0^{\mathcal{P}}$ denotes the subjective expectations operator, which we will address below in more detail. The utility function is of the usual iso-elastic form:

$$\mathbb{E}_0^{\mathcal{P}} \sum_{t=0}^{\infty} (\beta^i)^t U(c_t^i, n_t^i, h_t^i), \quad U(c_t^i, n_t^i, h_t^i) = \frac{\xi_c^i (c_t^i)^{1-\sigma}}{1-\sigma} + \frac{\xi_h^i (h_t^i)^{1-\nu}}{1-\nu} - \frac{\xi_n^i (n_t^i)^{1+\varphi}}{1+\varphi} \tag{1}$$

Households consume, borrow or lend in bonds, buy housing at the price q_t, which is subject to a quadratic adjustment cost, receive/pay interest on bonds, receive income, receive returns on their housing investment if they are savers, and finally receive taxes and transfers. The budget constraint reads:

$$c_t^i + b_{t+1}^i + q_t[h_t^i - (1-\delta^i)h_{t-1}^i] + \kappa_H^i (h_t^i - h_{t-1}^i)^2 + x_t^i = (1+r_t)b_t^i + w_t n_t^i + q_t f(x_t^i) + \Sigma_t^i + T_t^i \tag{2}$$

$f(x_t^i) = \xi_x \eta^{-1} x^\eta$ is the housing production function. We make the same assumption regarding profits and taxes as Bilbiie (2024). Only savers receive profits, which are then taxed and redistributed to borrowers. The taxation schedule is chosen such that countercyclical income risk arises, which is the empirically plausible case. Households are subject to a borrowing constraint which is given by:

$$-b_{t+1}^i \leq q_t h_t^i \phi_t \tag{3}$$

ϕ_t is exogenous and can be thought of as an exogenous loan to (housing) value constraint.

House price beliefs. Our formulation of house price beliefs follows the setup proposed in Roschitsch and Twieling (2024). As is standard in the literature on capital gains extrapolation (e.g., Adam and Marcet, 2011; Adam et al., 2017), households are endowed with a subjective probability measure over the entire sequence of variables they perceive as exogenous, henceforth referred to as external variables, denoted by $(r_t, w_t, \Sigma_t^i, T_t^i, \pi_t, q_t)_{t\geq 0}$. We denote this belief system by $\mathcal{P}$. The rational expectations benchmark corresponds to the special case in which this belief measure coincides with the objective (or "true"/equilibrium-implied) distribution over external variables, i.e., $\mathcal{P} = \mathbb{P}$.

While households may hold beliefs that deviate from the equilibrium-implied distribution, two key features are worth emphasizing. First, agents possess a time-consistent belief system. Second, they make decisions optimally given these beliefs. In this sense, agents are internally rational, as formalized by Adam and Marcet (2011).[5]

Conditional on the observed history of external variables up to period t, households use their belief system $\mathcal{P}$ to form expectations about the future evolution of these variables. We denote this subjective expectations operator by $\mathbb{E}_t^{\mathcal{P}}$, while the standard conditional rational expectations operator is denoted by $\mathbb{E}_t$. In our framework, agents are assumed to have rational expectations with respect to all external variables except for house prices, q_{t+s}.[6] Agents are assumed to entertain a simple state-space model for the evolution of house prices:

$$\begin{aligned} \ln \frac{q_{t+1}}{q_t} &= \ln m_{t+1} + \ln e_{t+1} \\ \ln m_{t+1} &= \varrho \ln m_t + \ln v_{t+1}, \quad \varrho \in (0,1) \\ \begin{pmatrix} \ln e_t & \ln v_t \end{pmatrix}' &\sim \mathcal{N}\left(\begin{pmatrix} -\frac{\sigma_e^2}{2} & -\frac{\sigma_v^2}{2} \end{pmatrix}, \begin{pmatrix} \sigma_e^2 & 0 \\ 0 & \sigma_v^2 \end{pmatrix} \right) \end{aligned} \tag{4}$$

[5] Moreover, it is not common knowledge among agents that all households share identical beliefs and preferences. As a consequence, agents are unable to uncover potential misspecifications in their beliefs—that is, the discrepancy $\mathcal{P} \neq \mathbb{P}$ cannot be resolved through deductive reasoning about the structure of the economy.

[6] Formally, $\mathcal{P} := \mathbb{P}_{-q} \otimes \mathcal{P}_q$, where $\mathbb{P}_{-q}$ is the objective measure over sequences of external variables without house prices, $\mathcal{P}_q$ is the measure over sequences of house prices implied by the described perceived model of house prices, and $\otimes$ is the product measure. Since we are interested in a first-order solution to the model, it does not matter what households perceive to be the dependence structure between house prices and the other external variables.

Equation (4) states that agents perceive house price growth rates to be the sum of a transitory and a persistent component. The disturbances $\ln e_t$ and $\ln v_t$ are not observable to the agents, rendering $\ln m_t$ unobservable. Agents apply the optimal Bayesian filter, i.e. the Kalman filter, to arrive at the observable system.[7]

Lemma 1 (House price belief updating)**.** *Applying the Kalman filter to the state-space model and log-linearizing around the non-stochastic steady-state gives:*

$$\mathbb{E}_t^{\mathcal{P}} \widehat{q}_{t+s} = \widehat{q}_t + \frac{1-\varrho^s}{1-\varrho} \varrho \widehat{\overline{m}}_t \tag{5}$$

$$\widehat{\overline{m}}_t = (\varrho - g)\widehat{\overline{m}}_{t-1} + g \Delta \widehat{q}_{t-1} \tag{6}$$

where $\ln \overline{m}_t := \mathbb{E}_t^{\mathcal{P}}(\ln m_t)$ *is the posterior mean,* $g = \frac{\sigma^2+\sigma_v^2}{\sigma^2+\sigma_v^2+\sigma_e^2}$ *is the steady-state Kalman filter gain,* $\sigma^2 = \frac{1}{2}[-\sigma_v^2 + \sqrt{\sigma_v^4 + 4\sigma_v^2\sigma_e^2}]$ *is the steady-state Kalman filter uncertainty, and* $\ln \widehat{e}_t$ *is perceived to be a white noise process.*

Proof. See Appendix A for the application of the Kalman filter. Log-linearization around the steady-state gives the result. ∎

Variables denoted with "$\widehat{\cdot}$" represent percentage deviations from their steady-state values. Equation (5) shows that expected future house prices depend on current prices and beliefs. The parameter $\varrho \in (0, 1)$ captures belief persistence, implying that the weight on prior beliefs increases with the forecast horizon. Current house prices enter one-to-one into expectations, reflecting extrapolative behavior. The belief-updating equation features an autoregressive component and adjusts based on observed past house price changes. Updating is more responsive when the Kalman gain g is high and belief persistence ϱ is low.

Firms and price setting. We assume a continuum of monopolistically competitive firms that produce intermediate good varieties and have the same beliefs as households. Firm beliefs, however, concern only variables over which households have rational expectations. Therefore, firms are rational. Firm j buys labor $n_t(j)$ from the representative labor packer and produces the variety $y_t(j)$ with a linear technology where labor is the only production factor. The firm sets its retail price $P_t(j)$ and maximizes the expected discounted stream of profits, subject to Rotemberg-type adjustment costs. The

[7] We assume agents' prior variance equals the steady-state Kalman variance.

log-linearized Phillips-Curve is given by:

$$\widehat{\pi}_t = \beta \mathbb{E}_t \widehat{\pi}_{t+1} + \frac{\epsilon - 1}{\kappa} \widehat{w}_t \tag{7}$$

Steady-state, Market clearing and Equilibrium. To solve the model, we take a first-order approximation around the non-stochastic steady-state. This steady state is equivalent under RE and SE. We ensure that the steady-state is efficient in terms of the production side by including a firm subsidy as is standard in the literature.

In equilibrium, labor, goods, and housing markets need to clear. Further, the monetary authority sets the nominal interest rate, i_t, according to a standard Taylor rule targeting only inflation, which includes a monetary policy (MP) shock ϵ_t^{mp}. The log-linearized formulation is given by:

$$\widehat{i}_t = \phi_\pi \widehat{\pi}_t + \widehat{\epsilon}_t^{MP}$$

Finally, the definition of the equilibrium is as follows:

Definition 1 (Internally Rational Expectations Equilibrium)**.** *An IREE consists of three bounded stochastic processes: shocks* $(\phi_t, \epsilon_t^{mp})_{t \geq 0}$*, allocations* $(c_t^i, b_t^i, h_t^i, n_t^i, x_t^s)_{i=s,h}$ *and prices* $(w_t, q_t, i_t, [P_t(j)]_{j \in [0,1]})$*, such that in all t*

1. *households choose* $(c_t^i, b_t^i, h_t^i, n_t^i, x_t^s)_{i=s,h}$ *optimally, given their beliefs* $\mathcal{P}$*,*
2. *firms choose* $([P_t(j)]_{j \in [0,1]})$ *optimally, given their beliefs* $\mathcal{P}$*,*
3. *the monetary authority acts according to a certain rule,*
4. *markets for consumption good varieties, hours, and housing clear given the prices.*

III. Results

We report results for three different model cases: first, the RANK case, second, the TANK case without fire sales, and third, the TANK case with fire sales.

III.A Model calibration

Before we turn to the results, we briefly discuss the calibration, which is summarized in table (1). Our calibration reflects a quarterly frequency, and we set the savers' discount factor accordingly. The HtM discount factor is chosen below the savers' discount factor to ensure that the borrowing constraint binds. The Frisch elasticity, the intertemporal elasticity of substitution, the housing utility elasticity, housing depreciation, the HtM share, and the Taylor rule coefficient are all standard parameter choices in line with the literature. We choose a steady state LTV ratio of 0.4. The LTV ratios on origination are between 0.8 and 1.0 (Greenwald, 2018). Our lower value reflects that not all HtM have mortgages and that some have been partially paid back. The taxation of saver profits is chosen such that the model generates countercyclical inequality as in Bilbiie (2024). Regarding the production side, we chose the price adjustment cost and elasticity of substitution across goods to target the slope of the Phillips Curve in Bilbiie (2024). The housing production elasticity is set as in Adam et al. (2022).

Table 1: Model parameters

Parameter		Value	Description	Source/ Target
Households	φ^i	1.500	inverse Frisch elasticity	standard
	σ^i	2.000	inverse of intertemporal EOS	standard
	ν^i	1.000	housing utility elasticity	Iacoviello (2005)
	δ^i	0.010	housing depreciation	1% quarterly depreciation
	β^s	0.995	discount factor	standard for quarterly frequency
	β^h	0.500	discount factor	sufficiently low for LTV to bind
	ϕ	0.400	steady state LTV	calibrated to match average LTV
	λ	0.350	HtM share	standard
	τ^d	0.350	Tax on firm profits	s.t. countercyclical inequality
Production	ϵ	6.000	EOS across varieties	slope of PC as in Bilbiie (2024)
	κ	250.00	price adjustment costs	
	η	0.800	elasticity of housing production	Adam et al. (2022)
Policy	ϕ_π	1.500	Taylor coefficient	standard

Notes: One period in the model is one quarter.

Concerning the steady state, we assume that there is no steady state inequality in consumption. This ensures that the aggregate housing stock, output, labor supply, and house prices are equivalent in the RANK and TANK model. Concerning housing, the

HtM hold 15% of housing in per capita terms in the steady state. Following Adam et al. (2022), the ratio of housing investment to aggregate consumption is 6%. The steady state wage is 1. The aggregate housing stock is roughly one quarter of annual output.

III.B The RANK case

We start with the standard RANK case. To do so, we simply set the share of the HtM to zero ($\lambda = 0$). We also assume that the housing adjustment cost is zero ($\kappa_H^s = 0$). It is important to note that the savers' block, described in this section, remains the same across the different model versions we present, and therefore the decompositions also remain valid.

Model solution. Solving this type of model under SE is not straightforward. Specifically, we have assumed that households do not have rational expectations about house prices and, in turn, provided them with a law of motion on how to form beliefs. At the same time, future household choices themselves are functions of the house price. Therefore, one needs to characterize these future expected internal variables as functions of future prices.[8] To solve the model, we employ the method developed in Roschitsch and Twieling (2024). In the following, we provide a brief summary: under SE, the consumption Euler equation takes the following form:

$$\widehat{c}_t^s = \sum_{s=0}^{\infty} \widehat{r}_{t+s} + \mathbb{E}_t^{\mathcal{P}} \widehat{c}_\infty \tag{8}$$

$\mathbb{E}_t^{\mathcal{P}} \widehat{c}_\infty$ is the subjective expectations wealth effect. It captures the household's expectations of future consumption in the far future as $s \to \infty$. Intuitively, assume that a transitory shock raises house prices today. Due to extrapolation, households believe that house prices will further rise in the future, which will lead them to believe that they will be able to consume more in the limit, hence $\mathbb{E}_t^{\mathcal{P}} \widehat{c}_\infty$ increases. Under RE this wealth effect is always zero, because agents understand that the economy will transition back to the steady state eventually.

It can be shown that $\mathbb{E}_t^{\mathcal{P}} \widehat{c}_\infty$ has an analytical formulation depending only on external variables. Furthermore, the knowledge of $\mathbb{E}_t^{\mathcal{P}} \widehat{c}_\infty$ is sufficient to characterize all internal variables in expectations. This representation therefore allows us to solve the model

[8] In our framework, $\mathbb{E}_t^{\mathcal{P}} \widehat{h}_{t+s}^s$ and $\mathbb{E}_t^{\mathcal{P}} \widehat{c}_{t+s}^s$ enter directly into the households' first-order conditions. As a result, it is necessary to explicitly characterize these expectations.

under SE. The system of equations representing the households' first-order conditions after applying the solution method is given in Appendix (B).

House price dynamics. Next, we illustrate the house price dynamics under RE and SE. Throughout the paper, we consider a 25 bp contractionary monetary policy shock in the SE model. What essentially matters for the model responses is the path of the real rate. To ensure that the RE and SE model are comparable, we always consider a 25 bp shock in the SE model, compute the path of the real rate, and then feed this path into the RE model.[9] This type of shock is well understood empirically and is well defined across all versions of our model. The house price response to this shock can be decomposed in the following way:

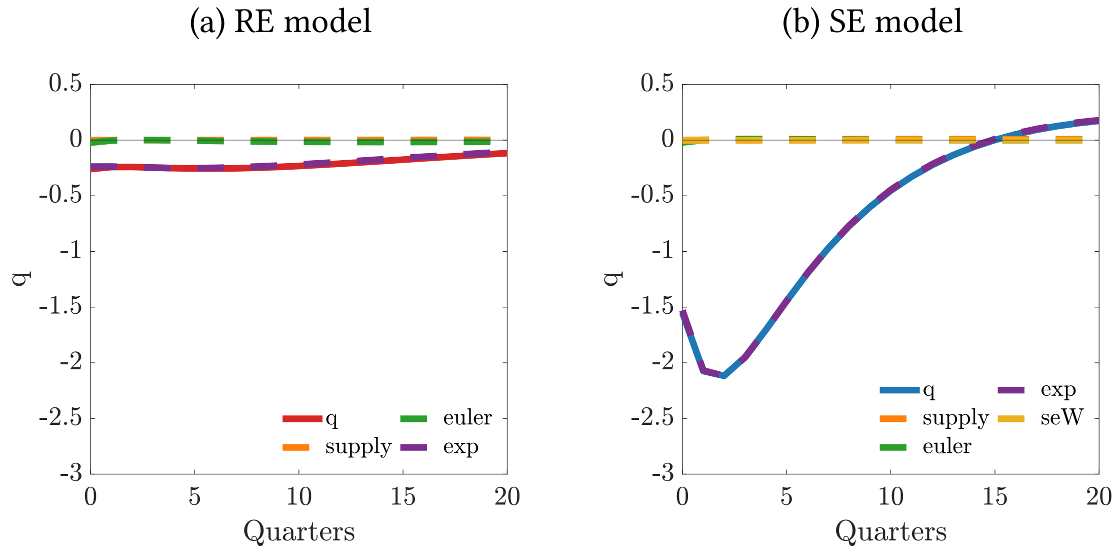
$$\widehat{q}_t = \underbrace{-\mathbb{E}_t\widehat{r}_{t+1} - (1-\bar{\beta})\sum_{n>0}\mathbb{E}_t\widehat{r}_{t+n+1}}_{Euler} - \underbrace{\bar{\nu}\widehat{h}^s_t}_{supply} + \underbrace{\bar{\beta}\mathbb{E}^{\mathcal{P}}_t\widehat{q}_{t+1}}_{expectations} + \underbrace{\sigma(1-\bar{\beta})\mathbb{E}^{\mathcal{P}}_t\widehat{c}_{\infty}}_{SE\ wealth} \tag{9}$$

This formulation is given by the housing Euler/demand equation from the household problem in which we have defined $\bar{\beta} = \beta^s(1-\delta)$ and $\bar{\nu} = \nu(1-\bar{\beta})$. The decomposition consists of four parts: First, an intertemporal or Euler part, which appears because housing is durable and can therefore be used for consumption smoothing. Second, a housing supply part, which states that a more flexible housing supply will lead to less volatility in house prices. Third, if the house prices are expected to increase in the future, that leads to an increase in the price today, reflecting an asset pricing component. And fourth, the subjective expectations wealth effect, which was already discussed above. This part will be zero under RE.

We now turn to the response of house prices to a contractionary monetary policy shock shown in Figure (1). One can observe that the response in the SE model is roughly four times larger than under RE. This is in line with a large literature documenting that belief extrapolation amplifies the volatility of asset prices. For both models, it also seems to be the case that house price expectations drive the majority of the response, although under RE, we observe a small response due to intertemporal substitution once accounting for the small overall responsiveness of the house price.

[9]This essentially means that in the RE model we are not considering a 25 bp shock but a sequence of shocks that generates the same path of the real rate as in the SE model.

Figure 1: House price response to MP shock, decomposition



Notes: Responses to contractionary MP shock (25 bp) in SE model. The real rate path is the same in the RE and SE model.

It seems surprising that almost all the variation in house prices is driven by expectations, especially since a large literature empirically documents significant effects from housing supply frictions on house prices.[10] Under RE it is indeed difficult to generate house price volatility through changes in the housing supply side. However, under SE, we can derive an alternative decomposition that does not exist under RE. Specifically, one can solve for the fixed point in the house price by substituting $\mathbb{E}^{\mathcal{P}}_t \widehat{q}_{t+1}$ and $\mathbb{E}^{\mathcal{P}}_t \widehat{c}_\infty$, which are both functions of the current house price. We arrive at the following decomposition:

$$\widehat{q}_t = \frac{1}{1-\bar{Q}}\Bigg[\underbrace{-(1-\bar{\beta})^{-1}\mathbb{E}_t\widehat{r}_{t+1} - \sum_{n>0}\mathbb{E}_t\widehat{r}_{t+n+1}}_{Euler} - \underbrace{\nu\widehat{h}^s_t}_{supply} + \underbrace{\frac{\bar{\beta}}{1-\bar{\beta}}\varrho\widehat{\bar{m}}_t}_{\substack{posterior\ mean\\ expectations}} + \underbrace{\sigma\mathbb{E}^{\mathcal{P}}_t\tilde{c}_\infty}_{SE\ wealth}\Bigg] \tag{10}$$

We observe that instead of house price expectations at $t+1$, we now have the posterior mean expectations. In addition, $\mathbb{E}^{\mathcal{P}}_t\tilde{c}_\infty$ denotes the effect of the SE wealth net of house prices. Importantly, this is still a function of the posterior mean expectations $\widehat{\bar{m}}_t$. Finally, the whole decomposition is scaled by $(1-\bar{Q})^{-1}$ with $\bar{Q} \in (0,1)$. Just by comparing both decompositions, one can see that in equation (10) housing supply is likely to play a larger

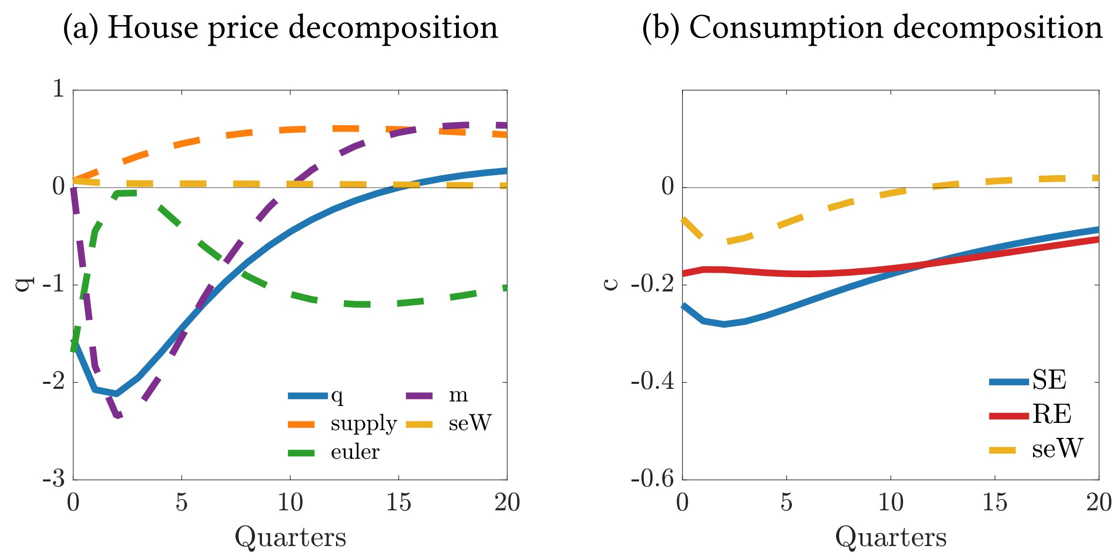
[10] See for instance: Mian et al. (2013); Aastveit and Anundsen (2022); Guren et al. (2021)

role: in equation (9) $\bar{\nu}$ scales the housing supply effect. As β is likely close to one and δ close to zero, $\bar{\nu}$ is fairly small. Under the alternative decomposition in equation (10), only ν appears, and therefore the coefficient scaling of housing supply is significantly larger.

This can be seen in panel (a) of Figure (2). Housing supply plays now a much larger role in balancing the negative response in the other channels, in particular in the long run. This is the case because it takes time for the housing stock to be built up. The posterior expectations respond with a lag to the shock, this is intuitive because they are a backward-looking function of house price growth. Further, the Euler effect is quite important, in particular on impact, which is due to the fact that the coefficient scaling the intertemporal substitution in equation (10) is significantly larger. Finally, the SE wealth effect again has only a marginal impact on house prices.

Our findings of both house price decompositions support the conclusion that the SE wealth effect generally plays a minor role. Panel (b) of Figure (2) shows the consumption decomposition derived in Equation (8). It is apparent that the SE wealth effect roughly accounts for one-third of the volatility in consumption for the first few quarters. Therefore, while this effect is negligible for house price responses, it is important for consumption dynamics.

Figure 2: House price response to MP shock, decomposition



Notes: Responses to contractionary MP shock (25 bp) in SE model. The real rate path is the same in the RE and SE model.

Output dynamics. Finally, we can focus on the output response. Given goods market clearing, output is just a function of consumption and investment:

$$\widehat{y}_t = \frac{c_{ss}}{y_{ss}}\widehat{c}_t^s + \frac{x_{ss}}{y_{ss}}\widehat{x}_t^s \tag{11}$$

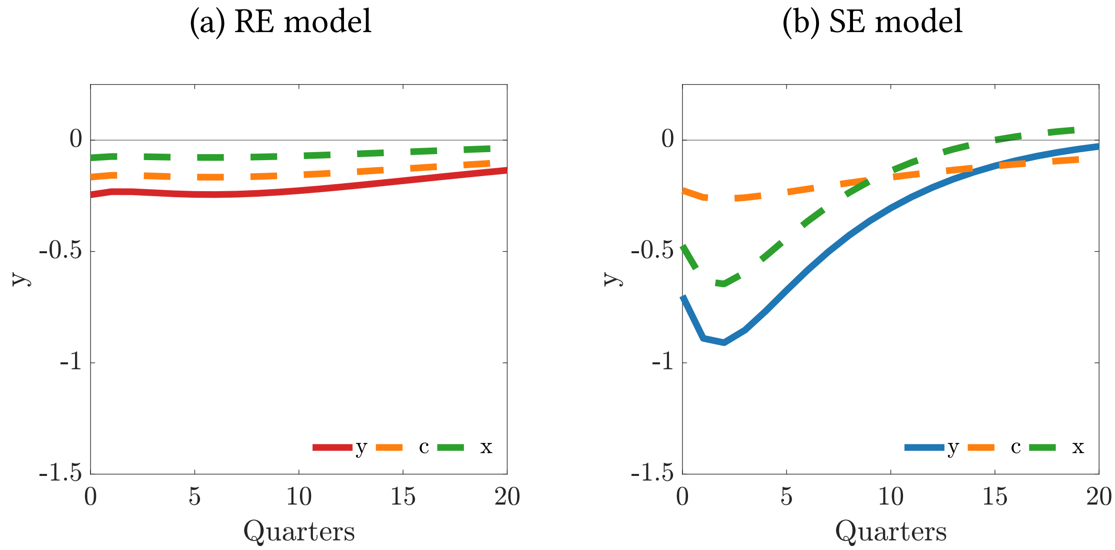
In Figure (3), we present the decomposition of the output response in both the RE and subjective expectations SE models. The overall output response in the SE framework is nearly four times larger than that observed under RE. This amplification is primarily attributable to the heightened responsiveness of housing investment. From the household optimization problem, we derive the following relationship:

$$\widehat{x}_t^s = (1-\eta)^{-1}\widehat{q}_t$$

which indicates that greater volatility in house prices translates directly into amplified fluctuations in housing investment. Consequently, under SE, housing investment emerges as the main driver of output dynamics, whereas under RE, the output response is predominantly driven by consumption. The results under SE are therefore consistent with the empirical evidence documented by Leamer (2007), who emphasizes the central role of housing in the business cycle. In contrast, the RE model fails to replicate this feature, underestimating the cyclical importance of housing. These findings highlight that under subjective expectations housing plays a significantly more prominent role in driving the business cycle.

It is also striking that the output response is quite persistent under RE. The reason is that the real rate response generated by the monetary policy shock under SE and which we feed into the RE model, is itself very persistent. Under RE, the consumption response is simply the sum of the future real interest rate path. If the real rate is persistent, so is the consumption response, and this spills over to the rest of the model.

Figure 3: Output response to MP shock, decomposition



Notes: Responses to contractionary MP shock (25 bp) in SE model. The real rate path is the same in the RE and SE model.

III.C The TANK case: collateral channel

We now turn to the TANK case of the model with only collateral effects being present. In this model formulation, the share of HtM is $\lambda = 0.35$. Further, we set the housing adjustment costs κ_H^i for savers to zero, and for the HtM we set $\kappa_H^h \to \infty$. This ensures that only the savers buy or sell housing units in response to shocks hitting the economy, while HtM agents stick with their steady state level of housing. Thereby, we can isolate the collateral effect on the HtM households.

The HtM consumption response is given by their budget constraint, in which we have substituted the labor supply, the borrowing constraint, and market clearing conditions to arrive at the following equation:

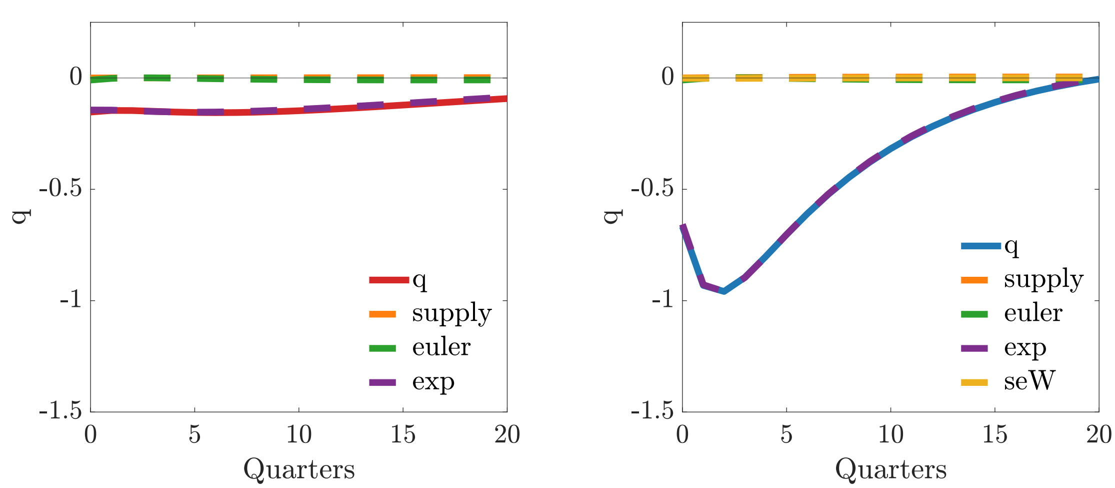
$$\widehat{c}_t^h = \underbrace{\chi_1^c \widehat{y}_t + \chi_2^c \widehat{x}_t}_{GE} + \underbrace{\chi_3^c \widehat{q}_t - \chi_4^c (\widehat{r}_t + \widehat{q}_{t-1})}_{Collateral(\phi)} - \underbrace{\chi_5^c \delta \widehat{q}_t}_{deprecation(h)} \tag{12}$$

The consumption behavior of HtM households can be decomposed into three components. First, a general equilibrium (GE) component operates through conventional channels: increasing aggregate output and housing investment raise labor demand and hence wages, thereby stimulating consumption. Since $\chi_1^c > 1$ inequality is countercyclical in the sense of Bilbiie (2024).[11] Second, a collateral channel captures the impact of fluctuations in house prices on borrowing constraints. When house prices rise, the relaxation of these constraints enables higher borrowing and thus higher consumption. Importantly, the level of debt incurred also influences future repayment obligations, and higher real interest rates similarly alter the repayment schedule, thus shaping HtM consumption. Finally, there exists a housing depreciation effect, which arises purely for accounting reasons. To maintain their steady state level of housing, HtM households must repurchase a fraction δ of housing each period to offset depreciation. Given the typically small magnitude of δ, this effect is quantitatively negligible.

House price dynamics. As before, we start by focusing on the house price dynamics, which we plot in Figure (4). The picture that emerges is similar to the RANK case: house prices are much more volatile under SE compared to RE. Again, the decomposition shows that most of the response is driven by house price expectations. It is important to note that the path of the real rate in the TANK model is not the same as in the RANK model. Therefore, one should be cautious with a quantitative comparison between the two models.

Figure 4: House price response to MP shock, decomposition

(a) RE model (b) SE model



Notes: Responses to contractionary MP shock (25 bp) in SE model. The real rate path is the same in the RE and SE model.

[11] For the remaining coefficients we have $\chi_j^c > 0, j \in \{2, 3, 4, 5\}$. For the definitions see Appendix (C).

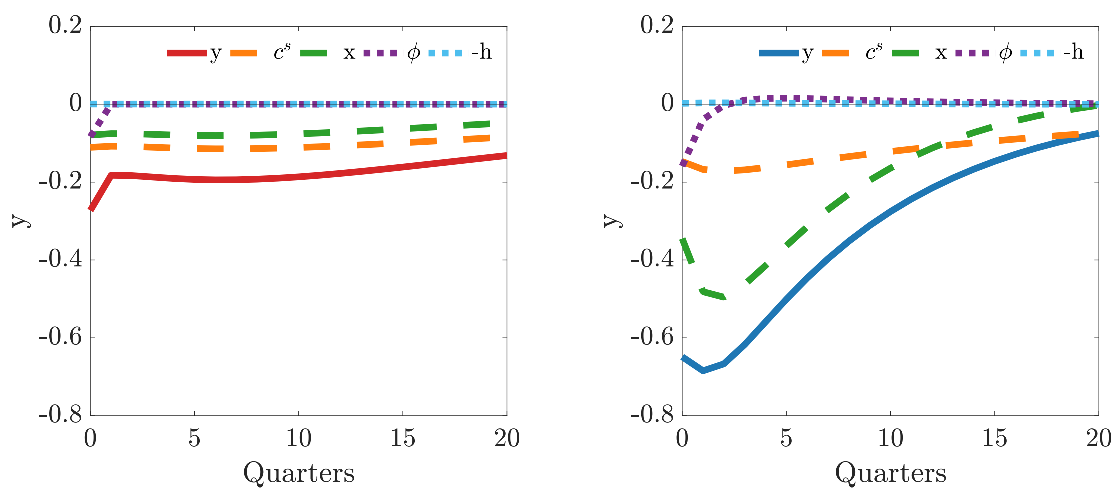
Output dynamics. In terms of output dynamics, we can derive a similar decomposition as in the RANK case. Equation (13) shows that it consists of saver consumption and housing investment, as in RANK. Additionally, the collateral channel and the depreciation parts now also show up.[12]

$$\widehat{y}_t = \chi_1^y \widehat{c}_t^s + \chi_2^y \widehat{x}_t + \underbrace{\chi_3^y \widehat{q}_t - \chi_4^y (\widehat{r}_t + \widehat{q}_{t-1})}_{Collateral(\phi)} - \underbrace{\chi_5^y \delta \widehat{q}_t}_{deprecation(h)} \tag{13}$$

Turning to the responses to the monetary policy shock, we show in Figure (5) that output is much more responsive under SE. As before, this is mainly driven by the stronger response in housing investment. Under RE, as before, saver consumption dominates the remaining channels. The collateral channel has only a marginal effect and is quite short-lived; this holds for both models. Housing depreciation has no impact, as discussed before.

Figure 5: Output response to MP shock, decomposition

(a) RE model (b) SE model



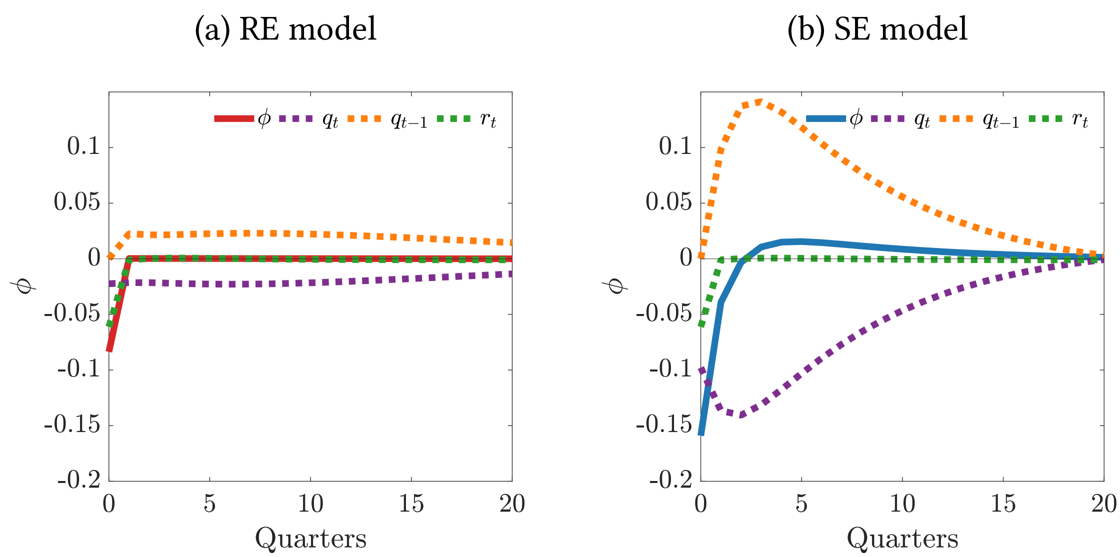
Notes: Responses to contractionary MP shock (25 bp) in SE model. The real rate path is the same in the RE and SE model.

Focusing on the collateral response, we decompose it in Figure (6) into its sub-components $\chi_3^y \widehat{q}_t$, $\chi_4^y \widehat{r}_t$, and $\chi_4^y \widehat{q}_{t-1}$. One can observe that the initial drop is driven by the increase in the real rate, $\widehat{i}_{t-1}$ is predetermined, but $\widehat{\pi}_t$ will respond, and the decline in the house prices. Following the initial drop, the collateral effect will quickly converge towards

[12]The coefficients are defined in Appendix (C). All coefficients are positive.

zero. That is because, as the HtM were able to borrow less initially, they will also have to repay less in the consecutive periods. In other words, as the real rate effect returns towards zero relatively quickly: $\chi_3^y \widehat{q}_t$, and $\chi_4^y \widehat{q}_{t-1}$ cancel each other out.[13] Under SE, the collateral response is roughly twice as strong on impact and is more persistent. As the real rate effect is the same in both models, the differential response needs to be driven by the stronger and more hump-shaped response of the house prices under SE.

Figure 6: Collateral decomposition in response to MP shock



Notes: Responses to contractionary MP shock (25 bp) in SE model. The real rate path is the same in the RE and SE model.

III.D The TANK case: fire sale channel

We now turn to our final channel, the fire sale channel. This channel occurs if HtM agents actively participate in the housing market. In our model, this means that $\kappa_H^h = 0$. We call this channel the fire sale channel because, under SE, house prices may fall below their fundamental value. If in this period HtM agents sell housing to stabilize the goods consumption, the sale takes place in an environment where housing is undervalued, and hence a fire sale occurs. Importantly, these sales may lead to a further decline in house prices, which could lead to further downward pressure on output through a decline in housing investment and a tightening of borrowing constraints.

[13] Algebraically this is also since $\chi_4^y = \beta^s \chi_3^y$, and therefore the coefficients are almost the same as β^s is close to one.

The budget constraint of the HtM, after several substitutions, is given by:

$$\widehat{c}_t^h = \underbrace{\chi_1^c \widehat{y}_t + \chi_2^c \widehat{x}_t}_{GE} + \underbrace{\chi_3^c(\widehat{q}_t + \widehat{h}_t) - \chi_4^c(\widehat{r}_t + \widehat{q}_{t-1} + \widehat{h}_{t-1})}_{Collateral(\phi)} - \underbrace{\chi_5^c(\delta \widehat{q}_t + \widehat{h}_t^h - (1-\delta)\widehat{h}_{t-1}^h)}_{fire\ sales(h)} \tag{14}$$

As in the TANK model without fire sales, the transmission mechanism operates through both a GE channel and a collateral channel. In the collateral channel, housing purchases now play an active role: increases in housing purchases expand the stock of collateral, thereby enhancing households' borrowing capacity. The final component is the fire sale channel, which enters with a negative sign. Specifically, when HtM households sell housing—i.e., when $\widehat{h}_t^h - (1-\delta)\widehat{h}_{t-1}^h$ declines—they are able to increase consumption by reallocating resources from housing to consumption goods.

Model solution. In a model in which the HtM are allowed to trade housing, we need to pin down the housing demand of these agents. As for the savers, this is done by the HtM housing Euler/demand equation (15). This equation is similar to the one from the savers, only that it now accounts for the binding LTV constraint. Here we have defined $\beta^\Delta = (\beta^s - \beta^h)/\beta^s$. Housing demand depends on consumption today and tomorrow, house prices today and tomorrow, as well as tomorrow's real rate.

$$\bar{\nu} h_t^h = \sigma(1-\phi)\widehat{c}_t^h - \sigma(\bar{\beta}^h - \phi(1-\beta^\Delta))\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+1}^h - (1-\phi\beta^\Delta)\widehat{q}_t + \bar{\beta}^h \mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+1} - \phi(1-\beta^\Delta)\mathbb{E}_t \widehat{r}_{t+1} \tag{15}$$

Crucially, the HtM housing Euler/demand equation (15) depends on future expected HtM consumption $\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+1}^h$. Under RE, this is not an issue, and we can continue in the standard way. But, as already discussed in the saver problem, under SE, we need to characterize future expected internal variables as a function of prices. For the saver, we heavily relied on the consumption Euler equation, which allowed us to characterize the SE wealth effect and thereby solve the model. In contrast to the saver, this is not possible for the HtM agent as the consumption Euler equation is not binding.

It is possible to characterize the future expected HtM consumption using the HtM budget constraint at $t+1$ in which we did not substitute for the market clearing conditions:

$$\begin{aligned}(c_{ss}^h + n_{ss}^h w_{ss}\sigma/\varphi)\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+1}^h = \Big(n_{ss}^h w_{ss} * (\frac{1}{\varphi}+1) - \frac{\tau^d}{\lambda}\Big)\mathbb{E}_t \widehat{w}_{t+1} + q_{ss}h_{ss}^h \phi(\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+1} + \mathbb{E}_t^{\mathcal{P}}\widehat{h}_{t+1}) \\ - \beta^{-1} q_{ss} h_{ss}^h \phi(\mathbb{E}_t \widehat{r}_{t+1} + \widehat{q}_t + \widehat{h}_t) - q_{ss}h_{ss}^h\Big(\delta \mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+1} + \mathbb{E}_t^{\mathcal{P}}\widehat{h}_{t+1}^h - (1-\delta)\widehat{h}_t^h\Big)\end{aligned} \tag{16}$$

However, this equation in turn depends on future expected housing choices of the HtM $\mathbb{E}_t^{\mathcal{P}}\widehat{h}_{t+1}^h$, which again depends on $\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+2}^h$. To solve this model, we therefore propose a method relying on lag polynomial factorization as outlined in Proposition (1).

Proposition 1 (Expected HtM consumption under Subjective Expectations). *For any s>1, the HtM budget constraint and the HtM housing Euler/demand equation can be presented in the following lag polynomial formulation:*

$$\wp(\mathbb{L})\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+s}^h = \mathbb{E}_t Z_{t+s} + Q(\mathbb{L})\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+s}$$

Where $\wp(\mathbb{L})$ and $Q(\mathbb{L})$ are lag polynomials of order two. $\mathbb{E}_t Z_{t+s}$ is a function of the prices over which the agent holds rational expectations. The solution of the lag polynomials yields the roots:

$$\gamma_1 := \frac{\mathrm{b}-\sqrt{\mathrm{b}^2-4\mathrm{ac}}}{2\mathrm{a}} \text{ and } \gamma_2 := \mathrm{c}/(\mathrm{a}\gamma_1).$$

Where a, b, c are the coefficients in $\wp(\mathbb{L})$. Further, the roots are real valued and $0 < \gamma_1 < 1$ and $\gamma_2 > 1$. Expected future consumption is therefore given by:

$$\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+s}^h = (\frac{\mathrm{a}\gamma_1}{\mathrm{c}})^s\widehat{c}_t^h + \sum_{n\geq 0}\gamma^n\sum_{k=0}^{s+n-1}[(\mathrm{a}\gamma)/\mathrm{c}]^k\left(\mathbb{E}_t Z_{t+s-k+n} + Q(\mathbb{L})\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+s-k+n}\right)$$

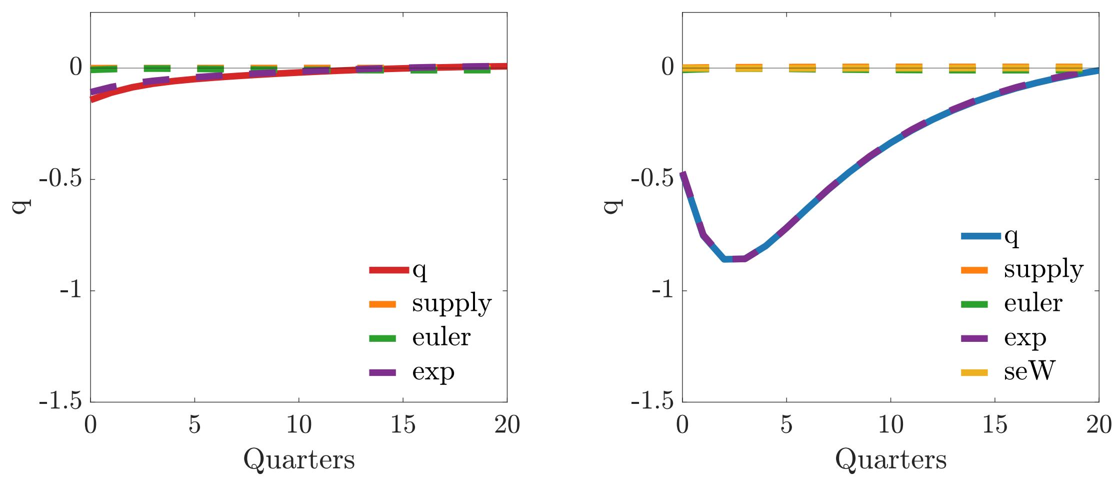
Proof. See Appendix (D). ∎

Proposition (1) illustrates that for any period $t+s$ and $s>1$ we can derive a representation of future expected HtM consumption that only depends on current HtM consumption $\widehat{c}_t^h$, future expected prices the households have rational expectations about, $\mathbb{E}_t Z_{t+s-k+n}$, and expectations about house prices $\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+s-k+n}$. Hence, we can derive a formulation of expected HtM consumption that only depends on current internal and expected external variables. Finally, the HtM choices are characterized by the HtM budget constraint at t and $t+1$, the HtM housing Euler/demand equation at t and $t+1$, as well as the HtM consumption expectations formulation derived in Proposition (1) at $t+2$ ($s=2$).

House price dynamics. We begin once again by examining the dynamics of house prices as depicted in Figure (7). Under SE, house price fluctuations are substantially amplified relative to RE. This amplification is primarily driven by the dynamics of house price expectations. These findings closely resemble the results obtained under the RANK and TANK formulation with only the collateral channel.

Figure 7: House price response to MP shock, decomposition

(a) RE model (b) SE model



Notes: Responses to contractionary MP shock (25 bp) in SE model. The real rate path is the same in the RE and SE model.

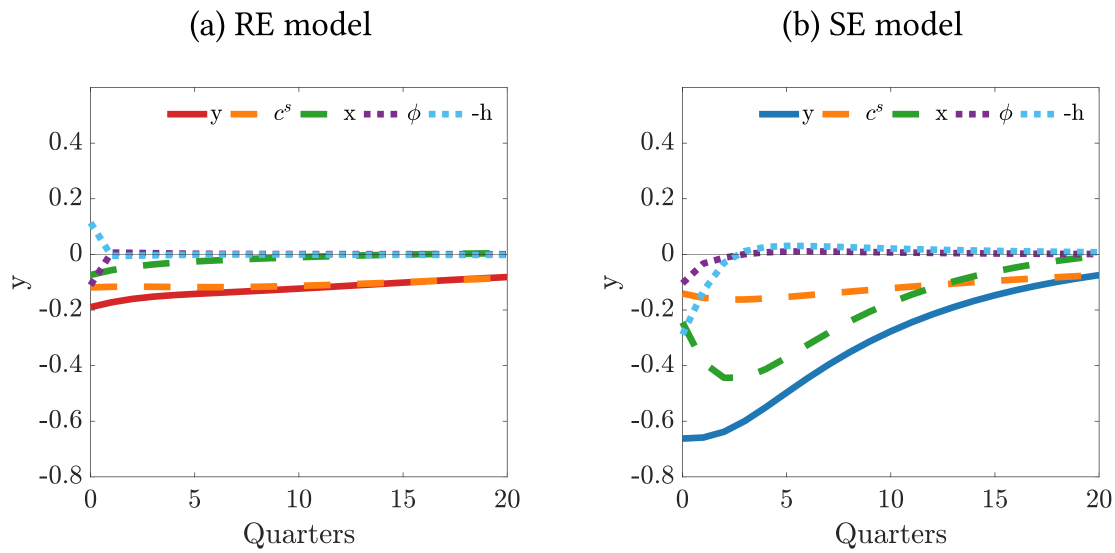
Output dynamics. Turning to the dynamics of output, the decomposition is presented in equation (17). In contrast to previous specifications, this decomposition now includes a fire-sale channel. Analogous to the decomposition of HtM consumption, housing sales by HtM households can serve as a stabilizing force for aggregate output. The intuition is that the HtM will use the proceeds of the sale to consume, which eventually stabilizes output.

$$\widehat{y}_t = \chi_1^y \widehat{c}_t^s + \chi_2^y \widehat{x}_t + \underbrace{\chi_3^y(\widehat{q}_t + \widehat{h}_t^h) - \chi_4^y(\widehat{r}_t + \widehat{q}_{t-1} + \widehat{h}_{t-1}^h)}_{Collateral(\phi)} - \underbrace{\chi_5^y(\delta\widehat{q}_t + \widehat{h}_t^h - (1-\delta)\widehat{h}_{t-1}^h)}_{fire\ sales(h)} \tag{17}$$

The output decomposition is shown in Figure (8). As before in the SE model, the output response is largely amplified. Similarly, in the SE model, this amplification is driven by housing investment while saver consumption is relatively muted. Further, under RE saver consumption dominates housing investment as already discussed above. The collateral channel, as before, operates only in the short term. Relative to the output response, it only plays a minor role under SE but a larger role under RE.

Turning to the fire sale channel, under RE, HtM households sell housing in order to stabilize consumption, thereby contributing to the stabilization of aggregate output. Moreover, the stronger impact of the collateral channel can also be attributed, at least in part, to these housing sales, which tighten borrowing constraints. Under SE the opposite is the case, the HtM buy housing, exerting downward pressure on output, but stabilizing house prices. In both cases, under RE and SE, the effect is relatively short-lived but relative to the output response sizable.

Figure 8: Output response to MP shock, decomposition

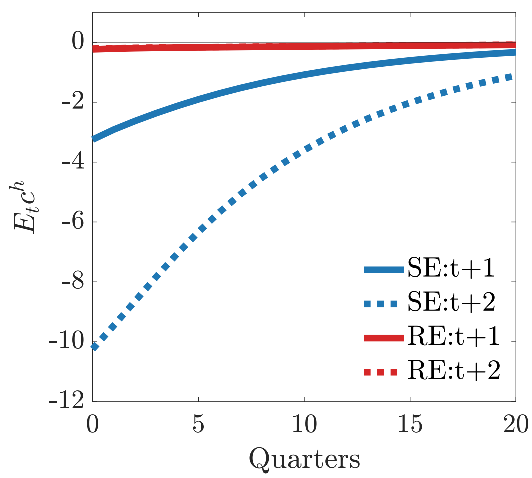


Notes: Responses to contractionary MP shock (25 bp) in SE model. The real rate path is the same in the RE and SE model.

The dynamics of the fire sale channel in the SE model seem surprising. One would expect the HtM, who have a high marginal propensity to consume (MPC), to sell housing in order to stabilize consumption and not the opposite. Based on equation (15), this type of behavior could be explained through one of the following dynamics: First, real rates strongly decrease, which we can rule out. Second, accounting for their relative coefficients, house price expectations react by less than the current house price. Third, again accounting for the relative coefficients, HtM consumption expectations drop by more than the current HtM consumption. The latter turns out to be the main driving force.

Figure (9) plots the consumption expectations one and two periods ahead. While under RE, the drop in both expectations is relatively small, under SE the drop is large for the $t+1$ expectations, but almost increases by a factor of four for expectations about $t+2$. This sharp decline in future expected consumption leads HtM households to buy housing to mitigate future consumption losses.

Figure 9: HtM consumption expectations in response to MP shock



Notes: Responses to contractionary MP shock (25 bp) in SE model. The real rate path is the same in the RE and SE model.

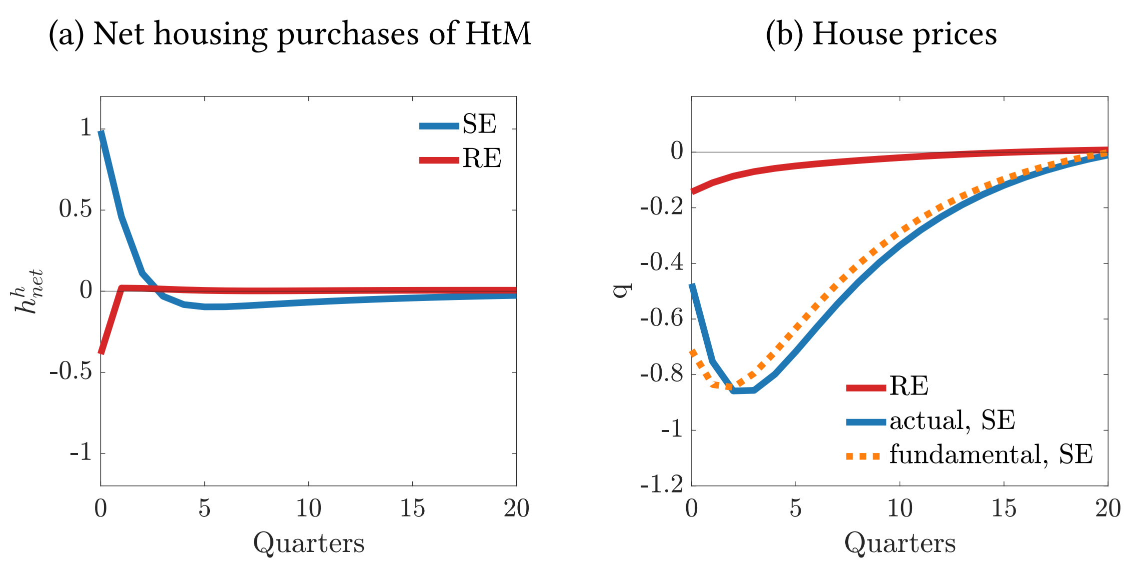
This raises the question of why HtM households' consumption expectations exhibit such pronounced sensitivity. Recall that Proposition (1) is used to derive $\mathbb{E}_t^{\mathcal{P}} \widehat{c}_{t+2}^h$, where the representation explicitly incorporates rational expectations about future wages, as captured by the term $\mathbb{E}_t Z_{t+s-k+n}$. Given the MPCs characteristic of HtM households, their consumption is highly responsive to fluctuations in income, both in realized and expected terms. As demonstrated in Appendix (D), the expression for $\mathbb{E}_t^{\mathcal{P}} \widehat{c}_{t+2}^h$ includes an infinite sum of rational expectations over future wages embedded in the $\mathbb{E}_t Z_{t+s-k+n}$ component. Due to the structure of the HtM environment, future wages receive substantial weight in this summation, thereby amplifying the sensitivity of consumption expectations.

To mitigate this excessive sensitivity, one could consider model extensions in which wage dynamics are less volatile, agents hold SE over future income, or additional constraints, such as payment-to-income limits.

Finally, we want to return to the notion of fire sales. In the SE model, it is theoretically possible to generate this phenomenon. To understand whether they occur in our example, we plot the HtM net purchases of housing in panel (a) of Figure (10). It shows the housing stock net of depreciation of the HtM, i.e., if it is positive, the HtM purchase housing relative to the steady state. Panel (b) plots the house prices, and for the SE model, the fundamental house prices. This is the house price that would arise if the value of housing is evaluated using the actual laws of motion, contrary to the perceived laws of motion.

Within this framework, a fire sale arises when HtM households engage in housing sales while the actual house price lies below its fundamental value. Turning to the figure, we observe that in the initial period under SE, HtM households purchase housing, despite the fundamental price being lower than the prevailing market price. This implies that HtM households are acquiring housing at overvalued prices. After approximately three periods, the behavior reverses: HtM households begin to sell housing while the fundamental price exceeds the actual price. It is during these periods that fire sales materialize, as housing is sold at a discount relative to its fundamental valuation.

Figure 10: Fire sales in response to MP shock



Notes: Responses to contractionary MP shock (25 bp) in SE model. The real rate path is the same in the RE and SE model.

IV. Conclusion

In this paper, we investigate the interaction between housing and business cycles within a tractable heterogeneous agent New Keynesian model with extrapolative house price beliefs. In this model, the interaction between housing and business cycles can be broken down into consumption, housing investment, collateral effects, and fire sales. We show that in the SE model, output is much more volatile, which is largely driven by residential investment. Under RE, consumption dominates housing investment in every model type we investigate. Finally, we propose a new method on how to solve TANK models with housing trade under extrapolative house price beliefs.

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Appendix

A. Proof of Lemma (1)

Agents apply the optimal Bayesian filter, i.e. the Kalman filter, to arrive at the observable system:[14]

$$\ln \frac{q_{t+1}}{q_t} = \varrho \ln \overline{m}_t + \ln \widehat{e}_{t+1}$$
$$\ln \overline{m}_t = \varrho \ln \overline{m}_{t-1} - \frac{\sigma_v^2}{2} + g \cdot \left(\ln \widehat{e}_t + \frac{\sigma_e^2 + \sigma_v^2}{2} \right)$$

where $\ln \overline{m}_t := \mathbb{E}_t^{\mathcal{P}}(\ln m_t)$ is the posterior mean, $g = \frac{\sigma^2+\sigma_v^2}{\sigma^2+\sigma_v^2+\sigma_e^2}$ is the steady-state Kalman filter gain, $\sigma^2 = \frac{1}{2}[-\sigma_v^2 + \sqrt{\sigma_v^4 + 4\sigma_v^2\sigma_e^2}]$ is the steady-state Kalman filter uncertainty, and $\ln \widehat{e}_t$ is perceived to be a white noise process.

To avoid simultaneity in the house price we modify the belief setup following Adam et al. (2017).[15] We obtain the same observable system but with lagged information being used in the posterior mean updating equation:

$$\ln \overline{m}_t = (\varrho - g) \left(\ln \overline{m}_{t-1} - \frac{\sigma_v^2}{2} \right) + g \left(\ln \frac{q_{t-1}}{q_{t-2}} + \frac{\sigma_e^2}{2} \right). \tag{A.1}$$

Under this formulation, the posterior mean is pre-determined. We may now derive the posterior mean on the $s > 0$ periods ahead of price:

$$\mathbb{E}_t^{\mathcal{P}} q_{t+s} = q_t \cdot \exp\left(\ln \overline{m}_t \cdot \varrho \frac{1-\varrho^s}{1-\varrho} + \frac{1}{2}\sigma^2 \left(\varrho \frac{1-\varrho^s}{1-\varrho}\right)^2 \right) \cdot \exp(V), \quad V \propto \sigma_v^2 \tag{A.2}$$

This completes the proof.

[14] We assume agents' prior variance equals the steady-state Kalman variance.

[15] q_t appears twice: in the forecast equation, and in the Kalman-updating Equation through $\ln \widehat{e}_t$. Since q_t depends on $\overline{m}_t$, but the latter also depends on the former, it is not assured that at any point an equilibrium asset price exists and whether it is unique. See Adam et al. (2017) for the details. The idea of the modification is to alter agents' perceived information setup in that they observe each period one component of the lagged transitory price growth.

B. Equilibrium equations saver block under SE

The saver's decisions at an arbitrary calendar date t up to first order around the deterministic steady state is:

$$
\begin{aligned}
&(h) \quad \widehat{h}_t = -\frac{\sigma}{\nu}\frac{\frac{1}{\sigma}\sum_{n\geq 1}\mathbb{E}_t\{\widehat{r}_{t+n}\} - \bar{\beta}\frac{1}{\sigma}\sum_{n\geq 1}\mathbb{E}_t\{\widehat{r}_{t+n+1}\}}{1-\bar{\beta}} - \frac{1}{\nu}\widehat{q}_t + \frac{1}{\nu}\frac{\bar{\beta}}{1-\bar{\beta}}\varrho\widehat{\bar{m}}_t + \frac{\sigma}{\nu}\mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty, \\
&(n) \quad \varphi\widehat{n}_t + \sigma\widehat{c}_t = \widehat{w}_t, \\
&(b) \quad \widehat{c}_t = -\frac{1}{\sigma}\sum_{n\geq 1}\mathbb{E}_t\{\widehat{r}_{t+n}\} + \mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty, \\
&(x) \quad \widehat{q}_t = (1-\eta)\widehat{x}_t,
\end{aligned}
\tag{B.1}
$$

For notational ease, we have dropped the superscript s in the savers' choices. The SE wealth effect is given by,

$$
\begin{aligned}
\mathbb{E}_t^{\mathcal{P}}\widehat{c}_\infty = {} & \frac{\delta q_{ss}h_{ss}/\nu}{c_{ss} + \sigma/\varphi \cdot n_{ss}w_{ss} + \delta q_{ss}h_{ss}\sigma/\nu} \cdot \left[\widehat{q}_t + \widehat{\bar{m}}_t \cdot \frac{\varrho}{1-\varrho}\underbrace{\left(1 + \frac{1-\bar{\beta}\varrho}{1-\bar{\beta}}\frac{1-\varrho-\delta}{1-\beta\varrho}\frac{1-\beta}{\delta}\right)}_{>0\ \forall \beta,\delta,\varrho \in (0,1)}\right] \\
& + \frac{y_{ss}}{c_{ss} + \sigma/\varphi \cdot n_{ss}w_{ss} + \delta q_{ss}h_{ss}\sigma/\nu}\frac{1-\beta}{\beta} \cdot \left[\sum_{n=1}^{\infty}\beta^n\mathbb{E}_t\{z^*_{t+n}\} + \widehat{b}_{t+1}\right].
\end{aligned}
$$

And we have defined the following auxiliary variables and assumed that the savers' transfers are zero outside of the steady state:

$$
\begin{aligned}
z^*_{t+s} &:= \frac{w_{ss}n_{ss}}{y_{ss}}(1+1/\varphi)\widehat{w}_{t+s} + \widehat{\Sigma}_{t+s} - \frac{c_{ss}+n_{ss}w_{ss}\sigma/\varphi}{y_{ss}}\widehat{c}^*_{t+s} - \frac{q_{ss}h_{ss}}{y_{ss}}[\widehat{h}^*_{t+s} - (1-\delta)\widehat{h}^*_{t+s-1}], \\
\widehat{h}^*_{t+s} &:= -\frac{\sigma}{\nu}\frac{\frac{1}{\sigma}\sum_{n\geq 1}\mathbb{E}_{t+s}\{\widehat{r}_{t+s+n}\} - \bar{\beta}\frac{1}{\sigma}\sum_{n\geq 1}\mathbb{E}_{t+s}\{\widehat{r}_{t+s+n+1}\}}{1-\bar{\beta}}, \\
\widehat{c}^*_{t+s} &:= -\frac{1}{\sigma}\sum_{n\geq 1}\mathbb{E}_{t+s}\{\widehat{r}_{t+s+n}\}
\end{aligned}
$$

C. Definition of coefficients for decompositions

In this section, we define the coefficients in the consumption and output decompositions. The decompositions are derived by substituting the labor supply equation, goods-, and labor market clearing into the budget constraint of the HtM and solving for consumption. All coefficients, for consumption and output, are positive under our calibration. The definitions for coefficients in the consumption decompositions are given by:

$$\chi_1^c = (1 + \varphi(n_{ss}^h w_{ss} - \frac{\tau^d}{\lambda})(\varphi + y_{ss}\sigma)/(\varphi + \sigma)$$

$$\chi_2^c = (1 - \varphi(n_{ss}^h w_{ss} - \frac{\tau^d}{\lambda})(1 - \lambda)\sigma x_{ss}^s$$

$$\chi_3^c = \frac{\varphi}{\varphi + \sigma} q_{ss} h_{ss}^h \phi$$

$$\chi_4^c = \frac{\varphi}{\varphi + \sigma} q_{ss} h_{ss}^h \phi / \beta^s$$

$$\chi_5^c = \frac{\varphi}{\varphi + \sigma} q_{ss} h_{ss}^h$$

The definitions for coefficients in the output decompositions are given by:

$$\chi_1^y = \frac{1 - \lambda}{\lambda} / (y_{ss}\lambda^{-1} - \chi_1^c)$$

$$\chi_2^y = (\lambda^{-1} - (n_{ss}^h w_{ss} - \frac{\tau^d}{\lambda})\varphi\sigma + \sigma)(1 - \lambda)x_{ss}^s / (y_{ss}\lambda^{-1} - \chi_1^c)$$

$$\chi_3^y = \chi_3^c / (y_{ss}\lambda^{-1} - \chi_1^c)$$

$$\chi_4^y = \chi_4^c / (y_{ss}\lambda^{-1} - \chi_1^c)$$

$$\chi_5^y = \chi_5^c / (y_{ss}\lambda^{-1} - \chi_1^c)$$

D. Proof of Proposition (1)

We have that for $s > 1$:

$$\bar{\nu}\mathbb{E}_t^{\mathcal{P}} h_{t+s}^h = \sigma(1 - \phi)\widehat{c}_{t+s}^h - \sigma(\bar{\beta} - \phi(1 - \beta^{\Delta}))\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+s+1}^h - (1 - \phi\beta^{\Delta})\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+s} + \bar{\beta}\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+s+1} - \phi(1 - \beta^{\Delta})\mathbb{E}_t\widehat{r}_{t+s+1}$$

$$(c_{ss}^h + n_{ss}^h w_{ss}\sigma/\varphi)\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+s}^h = \left(n_{ss}^h w_{ss} * (\frac{1}{\varphi} + 1) - \frac{\tau^d}{\lambda}\right)\mathbb{E}_t\widehat{w}_{t+s} + q_{ss}h_{ss}^h\phi(\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+s} + \mathbb{E}_t^{\mathcal{P}}\widehat{h}_{t+s})$$
$$- \beta^{-1}q_{ss}h_{ss}^h\phi(\mathbb{E}_t\widehat{r}_{t+s} + \mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+s-1} + \mathbb{E}_t^{\mathcal{P}}\widehat{h}_{t+s-1}) - q_{ss}h_{ss}^h\left(\delta\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+s} + \mathbb{E}_t^{\mathcal{P}}\widehat{h}_{t+s}^h - (1 - \delta)\mathbb{E}_t^{\mathcal{P}}\widehat{h}_{t+s-1}^h\right)$$

Plugging the housing demand equation into the HTM budget, rearranging, and expressing in terms of the lag operator yields:

$$\Big[c^h_{ss} + n^h_{ss} w_{ss}\sigma/\varphi + q_{ss}h^h_{ss}\bar{v}^{-1}\sigma((1-\phi)^2 + (\bar{\beta}-\phi)(1-\beta^{\Delta})(1-\delta-\phi/\beta))$$
$$- q_{ss}h^h_{ss}\bar{v}^{-1}\sigma(1-\phi)(\bar{\beta}-\phi)(1-\beta^{\Delta})\mathbb{L}^{-1} - q_{ss}h^h_{ss}\bar{v}^{-1}\sigma(1-\delta-\phi/\beta)(1-\phi)\mathbb{L}\Big]\mathbb{E}^{\mathcal{P}}_t\widehat{c}^h_{t+s} =$$
$$\Big(n^h_{ss}w_{ss}*(\frac{1}{\varphi}+1)-\frac{\tau^d}{\lambda}\Big)\mathbb{E}_t\widehat{w}_{t+s}-q_{ss}h^h_{ss}\phi\Big(\beta^{-1}+\bar{v}^{-1}(1-\beta^{\Delta})(1-\delta-\phi/\beta)\Big)\mathbb{E}_t\widehat{r}_{t+s}+q_{ss}h^h_{ss}\phi\bar{v}^{-1}(1-\phi)(1-\beta^{\Delta})\mathbb{E}_t\widehat{r}_{t+s+1}$$
$$+ q_{ss}h^h_{ss}\Big[\phi-\delta+\bar{v}^{-1}\{(1-\phi)(1-\phi\beta^{\Delta})+\bar{\beta}(1-\delta-\phi/\beta)\}$$
$$+\{\phi/\beta+\bar{v}^{-1}(1-\beta^{\Delta}\phi)(1-\delta-\phi/\beta)\}\mathbb{L}-\bar{v}^{-1}\bar{\beta}(1-\phi)\mathbb{L}^{-1}\Big]\mathbb{E}^{\mathcal{P}}_t\widehat{q}_{t+s}$$

Hence, the equation has the following form:

$$\wp(\mathbb{L})\mathbb{E}^{\mathcal{P}}_t\widehat{c}^h_{t+s} = \mathbb{E}_t Z_{t+s} + Q(\mathbb{L})\mathbb{E}^{\mathcal{P}}_t\widehat{q}_{t+s}$$

We can now define:

$$\wp : \mathbb{C}\to\mathbb{C} : z \mapsto -\mathrm{a}z + \mathrm{b} - \mathrm{c}z^{-1}$$
$$\mathrm{a} := q_{ss}h^h_{ss}\bar{v}^{-1}\sigma(1-\delta-\phi/\beta)(1-\phi)$$
$$\mathrm{b} := c^h_{ss} + n^h_{ss}w_{ss}\sigma/\varphi + q_{ss}h^h_{ss}\bar{v}^{-1}\sigma((1-\phi)^2+(\bar{\beta}-\phi)(1-\beta^{\Delta})(1-\delta-\phi/\beta))$$
$$\mathrm{c} := q_{ss}h^h_{ss}\bar{v}^{-1}\sigma(1-\phi)(\bar{\beta}-\phi)(1-\beta^{\Delta})$$
$$\mathbb{E}_t Z_{t+s} := \chi^z_1\mathbb{E}_t\widehat{w}_{t+s} - \chi^z_2\mathbb{E}_t\widehat{r}_{t+s} + \chi^z_3\mathbb{E}_t\widehat{r}_{t+s+1}$$
$$Q : \mathbb{C}\to\mathbb{C} : z \mapsto q_{ss}h^h_{ss}\Big[\chi^m_1 + \chi^m_2 z - \chi^m_3 z^{-1}\Big]$$

The polynomial $z \mapsto -\wp(z)z$ has the roots

$$\gamma_1 := \frac{\mathrm{b}-\sqrt{\mathrm{b}^2-4\mathrm{ac}}}{2\mathrm{a}} \text{ and } \gamma_2 := \mathrm{c}/(\mathrm{a}\gamma_1).$$

One can show that $\mathrm{b}^2 - 4\mathrm{ac} > 0$ and therefore the roots are real valued. Also, given realistic calibrations of the model, $0 < \gamma_1 < 1$ and $\gamma_2 > 1$. Now we can factorize:

$$-\wp(z)z = \mathrm{c}(1 - z/\gamma_1)\,(1 - z\cdot(\mathrm{a}\gamma_1)/\mathrm{c})$$

so that

$$\wp(z) = \mathrm{c}/\gamma_1(1 - z^{-1}\gamma_1)\,(1 - z\cdot(\mathrm{a}\gamma_1)/\mathrm{c})$$

And we finally get:

$$\wp^{-1}(z) = \gamma_1/\mathrm{c}\left(\sum_{s\geq 0}\gamma_1^s z^{-s}\right)\left(\sum_{s\geq 0}((\mathrm{a}\gamma_1)/\mathrm{c})^s z^s\right)$$

Now we can apply this to our expected consumption formulation

$$\begin{aligned}
&\wp(\mathbb{L})\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+s}^h = \mathbb{E}_t Z_{t+s} + Q(\mathbb{L})\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+s} \\
\iff &\frac{\mathrm{c}}{\gamma_1}(1-\gamma_1\mathbb{L}^{-1})\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+s}^h = \frac{\mathrm{a}\gamma_1}{\mathrm{c}}\frac{\mathrm{c}}{\gamma_1}(1-\gamma_1\mathbb{L}^{-1})\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+s-1}^h + \mathbb{E}_t Z_{t+s} + Q(\mathbb{L})\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+s} \\
\iff &\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+s}^h = (\frac{\mathrm{a}\gamma_1}{\mathrm{c}})^s\widehat{c}_t^h + \sum_{n\geq 0}\gamma^n\sum_{k=0}^{s+n-1}\left[(\mathrm{a}\gamma)/\mathrm{c}\right]^k\left(\mathbb{E}_t Z_{t+s-k+n} + Q(\mathbb{L})\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+s-k+n}\right)
\end{aligned}$$

Finally, the following system of equations determines the choices of the HTM agent:

$$\bar{\nu}h_t^h = \sigma(1-\phi)\widehat{c}_t^h - (\sigma\bar{\beta} - \phi(1-\beta^\Delta))\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+1}^h - (1-\phi\beta^\Delta)\widehat{q}_t + \bar{\beta}\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+1} - \phi(1-\beta^\Delta)\mathbb{E}_t\widehat{r}_{t+1}$$

$$\bar{\nu}\mathbb{E}_t^{\mathcal{P}}\widehat{h}_{t+1}^h = \sigma(1-\phi)\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+1}^h - (\sigma\bar{\beta}-\phi(1-\beta^\Delta))\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+2}^h - (1-\phi\beta^\Delta)\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+1} + \bar{\beta}\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+2} - \phi(1-\beta^\Delta)\mathbb{E}_t\widehat{r}_{t+2}$$

$$\widehat{c}_t^h = \chi_1^c\widehat{y}_t + \chi_2^c\widehat{x}_t + \chi_3^c(\widehat{q}_t + \widehat{h}_t) - \chi_3^c(\widehat{r}_t + \widehat{q}_{t-1} + \widehat{h}_{t-1}) - \chi_5^c(\delta\widehat{q}_t + \widehat{h}_t^h - (1-\delta)\widehat{h}_{t-1}^h)$$

$$\begin{aligned}
(c_{ss}^h + n_{ss}^h w_{ss}\sigma/\varphi)\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+1}^h = &\left(n_{ss}^h w_{ss} * (\frac{1}{\varphi}+1) - \frac{\tau^d}{\lambda}\right)\mathbb{E}_t\widehat{w}_{t+1} + q_{ss}h_{ss}^h\phi(\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+1} + \mathbb{E}_t^{\mathcal{P}}\widehat{h}_{t+1}) \\
&- \beta^{-1}q_{ss}h_{ss}^h\phi(\mathbb{E}_t\widehat{r}_{t+1} + \widehat{q}_t + \widehat{h}_t) - q_{ss}h_{ss}^h\left(\delta\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+1} + \mathbb{E}_t^{\mathcal{P}}\widehat{h}_{t+1}^h - (1-\delta)\widehat{h}_t^h\right)
\end{aligned}$$

$$\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+2}^h = (\frac{\mathrm{a}\gamma_1}{\mathrm{c}})^2\widehat{c}_t^h + \sum_{n\geq 0}\gamma_1^n\sum_{k=0}^{n+1}\left[(\mathrm{a}\gamma_1)/\mathrm{c}\right]^k\left(\mathbb{E}_t Z_{t+2-k+n} + Q(\mathbb{L})\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+2-k+n}\right)$$

Now first derive $\sum_{n\geq 0}\gamma_1^n\sum_{k=0}^{n+1}\left[(\mathrm{a}\gamma_1)/\mathrm{c}\right]^k\mathbb{E}_t Z_{t+2-k+n}$

For $n=0$: $\mathbb{E}_t Z_{t+2} + \frac{\mathrm{a}\gamma_1}{\mathrm{c}}\mathbb{E}_t Z_{t+1}$
For $n=1$: $\gamma_1\left(\mathbb{E}_t Z_{t+3} + \frac{\mathrm{a}\gamma_1}{\mathrm{c}}\mathbb{E}_t Z_{t+2} + (\frac{\mathrm{a}\gamma_1}{\mathrm{c}})^2\mathbb{E}_t Z_{t+1}\right)$
For $n=2$: $\gamma_1^2\left(\mathbb{E}_t Z_{t+4} + \frac{\mathrm{a}\gamma_1}{\mathrm{c}}\mathbb{E}_t Z_{t+3} + (\frac{\mathrm{a}\gamma_1}{\mathrm{c}})^2\mathbb{E}_t Z_{t+2} + (\frac{\mathrm{a}\gamma_1}{\mathrm{c}})^3\mathbb{E}_t Z_{t+1}\right)$
...

Hence, we get

$$\frac{\mathrm{a}\gamma_1}{\mathrm{c}} \times \sum_{n\geq 0}\gamma_1^n(\frac{\mathrm{a}\gamma_1}{\mathrm{c}})^n \times \mathbb{E}_t Z_{t+1} + \sum_{n\geq 0}\gamma_1^n(\frac{\mathrm{a}\gamma_1}{\mathrm{c}})^n \times \mathbb{E}_t Z_{t+2} + \gamma_1\sum_{n\geq 0}\gamma_1^n(\frac{\mathrm{a}\gamma_1}{\mathrm{c}})^n \times \mathbb{E}_t Z_{t+3} + \ldots$$

And finally, we get:

$$\sum_{n\geq 0}\gamma_1^n\sum_{k=0}^{n+1}[(\mathrm{a}\gamma_1)/\mathrm{c}]^k\,\mathbb{E}_t Z_{t+2-k+n} = \mathbb{E}_t\tilde{Z}_t + \frac{\mathrm{a}\gamma_1}{\mathrm{c}}(1-\frac{\mathrm{a}\gamma_1^2}{\mathrm{c}})^{-1}\mathbb{E}_t Z_{t+1}$$

$$\mathbb{E}_t\tilde{Z}_t = \gamma_1\mathbb{E}_t\tilde{Z}_{t+1} + (1-\frac{\mathrm{a}\gamma_1^2}{\mathrm{c}})^{-1}\mathbb{E}_t Z_{t+2}$$

Now consider $\sum_{n\geq 0}\gamma_1^n\sum_{k=0}^{n+1}[(\mathrm{a}\gamma_1)/\mathrm{c}]^k\,Q(\mathbb{L})\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+2-k+n}$

Note that $\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+s} = \widehat{q}_t + \frac{1-\varrho^s}{1-\varrho}\widehat{m}_t$. So we have:

$$\sum_{n\geq 0}\gamma_1^n\sum_{k=0}^{n+1}[(\mathrm{a}\gamma_1)/\mathrm{c}]^k\,q_{ss}h_{ss}^h\Big[(\bar{\phi}-\delta)+\bar{\nu}^{-1}(1+(1-\delta)\bar{\beta})-(\beta^{-1}\bar{\phi}+\bar{\nu}^{-1}(1-\delta))\mathbb{L}-\bar{\nu}^{-1}\bar{\beta}\mathbb{L}^{-1}\Big]\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+2-k+n}$$

We therefore get:

$$\sum_{n\geq 0}\gamma_1^n\sum_{k=0}^{n+1}[(\mathrm{a}\gamma_1)/\mathrm{c}]^k\,q_{ss}h_{ss}^h\Big[\chi_1^m-\chi_2^m-\chi_3^m\Big]\widehat{q}_t$$
$$+\sum_{n\geq 0}\gamma_1^n\sum_{k=0}^{n+1}[(\mathrm{a}\gamma_1)/\mathrm{c}]^k\,q_{ss}h_{ss}^h\tfrac{\varrho}{1-\varrho}\Big[\chi_1^m-\chi_2^m-\chi_3^m\Big]\widehat{m}_t$$
$$-\sum_{n\geq 0}\gamma_1^n\sum_{k=0}^{n+1}[(\mathrm{a}\gamma_1)/\mathrm{c}]^k\,q_{ss}h_{ss}^h\tfrac{\varrho}{1-\varrho}\Big[\chi_1^m\varrho^{2-k+n}-\chi_2^m\varrho^{1-k+n}-\chi_3^m\varrho^{3-k+n}\Big]\widehat{m}_t$$

First, one can show that

$$\sum_{n\geq 0}\gamma_1^n\sum_{k=0}^{n+1}[(\mathrm{a}\gamma_1)/\mathrm{c}]^k = \sum_{n\geq 0}\gamma_1^n\times\sum_{n\geq 0}[(\mathrm{a}\gamma_1)/\mathrm{c}]^n = \frac{1}{1-\gamma_1}\frac{1}{1-\mathrm{a}\gamma_1/\mathrm{c}}$$

Further, we have that

$$\sum_{n\geq 0}\gamma_1^n\sum_{k=0}^{n+1}[(\mathrm{a}\gamma_1)/\mathrm{c}]^k\,q_{ss}h_{ss}^h\tfrac{\varrho}{1-\varrho}\Big[\chi_1^m\varrho^{2-k+n}-\chi_2^m\varrho^{1-k+n}-\chi_3^m\varrho^{3-k+n}\Big]=$$
$$q_{ss}h_{ss}^h\tfrac{\varrho}{1-\varrho}\Big\{[(\mathrm{a}\gamma_1)/\mathrm{c}]\sum_{n\geq 0}\gamma_1^n[(\mathrm{a}\gamma_1)/\mathrm{c}]^n\Big[\chi_1^m\varrho^1-\chi_2^m\varrho^0-\chi_3^m\varrho^2\Big]$$
$$+\sum_{n\geq 0}\gamma_1^n[(\mathrm{a}\gamma_1)/\mathrm{c}]^n\times\sum_{j\geq 0}\gamma_1^j\varrho^j\times\Big[\chi_1^m\varrho^2-\chi_2^m\varrho^1-\chi_3^m\varrho^3\Big]\Big\}=$$
$$q_{ss}h_{ss}^h\tfrac{\varrho}{1-\varrho}\frac{1}{1-\mathrm{a}\gamma_1^2/\mathrm{c}}(\frac{1}{1-\varrho\gamma_1}+\frac{\mathrm{a}\gamma_1}{\mathrm{c}\varrho})\Big[\chi_1^m\varrho^2-\chi_2^m\varrho^1-\chi_3^m\varrho^3\Big])$$

Therefore, we finally have that:

$$
\begin{aligned}
&\sum_{n\geq 0}\gamma_1^n\sum_{k=0}^{n+1}[(\mathrm{a}\gamma_1)/\mathrm{c}]^k\,Q(\mathbb{L})\mathbb{E}_t^{\mathcal{P}}\widehat{q}_{t+2-k+n} = \\
&\frac{1}{1-\gamma_1}\frac{1}{1-\mathrm{a}\gamma_1/\mathrm{c}}q_{ss}h_{ss}^h\Big[\chi_1^m-\chi_2^m-\chi_3^m\Big]\widehat{q}_t \\
&+\frac{1}{1-\gamma_1}\frac{1}{1-\mathrm{a}\gamma_1/\mathrm{c}}q_{ss}h_{ss}^h\tfrac{\varrho}{1-\varrho}\Big[\chi_1^m-\chi_2^m-\chi_3^m\Big]\widehat{m}_t \\
&-\frac{1}{1-\mathrm{a}\gamma_1^2/\mathrm{c}}\Big(\frac{1}{1-\varrho\gamma_1}+\frac{\mathrm{a}\gamma_1}{\mathrm{c}\varrho}\Big)q_{ss}h_{ss}^h\tfrac{\varrho}{1-\varrho}\Big[\chi_1^m\varrho^2-\chi_2^m\varrho^1-\chi_3^m\varrho^3\Big]\widehat{m}_t
\end{aligned}
$$